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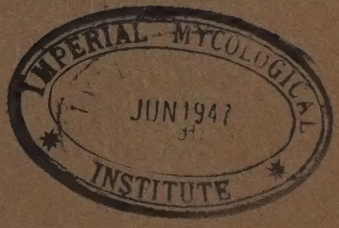

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MATHÉMATIQUES

NOUVELLES GÉNÉRALISATIONS DU THÉORÈME DE LIOUVILLE ET SON EXTENSION AUX ÉQUATIONS DU TYPE PARABOLIQUE

Par S. BERNSTEIN, de l'Académie

1. J'ai démontré dans mon travail [1] le théorème suivant:
Si la courbure de la surface

$$z = f(x, y), \tag{1}$$

où $f(x, y)$ est continue ainsi que ses dérivées des deux premiers ordres pour toutes les valeurs de (x, y), est négative en un point, où $p^2+q^2=A^2>0$, et s'il existe une grandeur $N_1>A^2$ telle que la courbure de la surface ne devient positive en aucun point, où $p^2+q^2\leqslant N_1$; dans ces conditions quel que soit le nombre donné h, il existe un nombre $\delta>0$ tel que l'inégalité

$$|z|<\sqrt{\delta^2(x^2+y^2)+h^2} \tag{2}$$

ne peut pas être réalisée en tous les points de la surface.

L'impossibilité de l'inégalité (2) sur toute la surface signifie que l'on n'a pas

$$\lim \frac{z}{\rho}=0 \qquad \text{pour} \quad \rho^2=x^2+y^2\to\infty \tag{2 bis}$$

c'est-à-dire que *la croissance de z* à l'infini est *d'ordre non inférieur à un.*

Soit P la limite inférieure de p^2+q^2 aux points de la surface (1), où la courbure est *positive*, et soit L la limite inférieure de p^2+q^2 aux points, où la courbure de la surface est *négative*. Cela étant, notre théorème peut s'énoncer sous la forme abrégée suivante:

La croissance à l'infini de la surface (1) *ne peut pas être d'ordre inférieur à un, si*

$$L<P. \tag{3}$$

En effet, $2\varepsilon<P-L$ étant suffisamment petit, il existera un point de la surface (1), où $rt-s^2<0$ et $p^2+q^2=L+\varepsilon=A^2>0$; mais en aucun point de la surface on n'aura en même temps $rt-s^2>0$ et $p^2+q^2\leqslant A^2+\varepsilon=L+2\varepsilon<P$.

2. Théorème I. *Si l'ordre de croissance de la surface* S est inférieur à un, s'il existe un nombre $P>0$ tel que dans toutes les régions S_P de la surface S, où*

* Ici, comme dans la suite, on admet toujours que la fonction $f(x, y)$ dans l'équation (1) de la surface S est continue, ainsi que ses dérivées des deux premiers ordres.

$$p^2+q^2<P \tag{4}$$

celle-ci satisfait à l'équation

$$A(x, y, z; p, q, r, s, t)\,r+2B(x, y, z, p, q, r; s; t)\,s+ \\ +C(x, y, z, p, q; r, s, t)\,t=0 \tag{5}$$

et à l'inégalité

$$AC-B^2>0 \tag{6}$$

ou à l'égalité

$$AC-B^2=0, \tag{7}$$

l'ensemble de points satisfaisant à (7), *ainsi que celui des points de discontinuité de A, B, C n'étant nullepart dense sur* S_P, *dans ces conditions la surface S se réduit à un plan horizontal.*

En effet, it résulte de (5) que

$$\begin{aligned}(Ar+Bs)^2+AC(rt-s^2)+(AC-B^2)s^2=0,\\ (Ct+Bs)^2+AC(rt-s^2)+(AC-B^2)s^2=0.\end{aligned} \tag{8}$$

Donc, en tout point où $AC-B^2>0$, on a $rt-s^2<0$, ou bien $s=r=t=0$; la première supposition est inadmissible, car elle entraîne la condition (3), qui est incompatible avec l'ordre de croissance inférieur à un de la surface. Les égalités

$$r=s=t=0 \tag{9}$$

ont donc lieu en tous les points d'un ensemble partout dense sur S_P, donc en tous les points, où

$$p^2+q^2<P. \tag{4}$$

Par conséquent, l'inégalité (4) est-elle ou bien satisfaite sur toute la surface ou bien elle n'a lieu nullepart. La seconde supposition étant incompatible avec la croissance d'ordre inférieur à un de la surface, il résulte de (9) que

$$z=p_0x+q_0y+C$$

est un plan qui doit être horizontal ($p_0=q_0=0$).

Indiquons à titre d'exemple les solutions de l'équation

$$y^2(1-pq)\,r+e^{\frac{y}{x}}t=0$$

qui pour $p^2+q^2<2$ satisfait à la condition

$$AC-B^2=y^2(1-pq)\,e^{\frac{y}{x}}\geqslant 0, \quad (x\gtrless 0),$$

l'égalité $AC-B^2=0$ n'ayant lieu que pour $y=0$.

3. Le théorème de Liouville sous la forme générale que nous venons de lui donner n'est pas applicable aux équations du type *parabolique*; dans ce cas il faut faire usage d'une forme *affaiblie* du théorème, où le plan horizontal est remplacé par les surfaces *cylindriques* à génératrices horizontales.

Théorème II. *Soit S une surface à croissance d'ordre inférieur à un, satisfaisant pour toutes les valeurs de* (x, y) *à l'équation* (5) *et à la condition* (6) *ou* (7) (cette fois l'égalité (7) peut même avoir lieu partout). *Dans ces conditions la surface S est une surface cylindrique à génératrices horizontales* (on suppose; bien entendu, que l'ensemble de points, où $A=B=C=0$, n'est nullepart dense sur la surface).

Il résulte, en effet, de (8) que l'on a

$$rt-s^2=0 \tag{10}$$

sur toute la surface. Par conséquent, la surface développable S est une surface cylindrique à génératrices horizontales.

Cependant, le plus souvent, comme il est facile de déterminer dans chaque cas particulier, les surfaces cylindriques bornées satisfaisant à l'équation (5) se réduisent à des plans horizontaux. Considérons, par exemple, l'équation parabolique linéaire à coefficients continus

$$A(x, y)\, r + 2B(x, y)\, s + C(x, y)\, t = 0 \tag{11}$$

$$AC - B^2 = 0. \tag{7}$$

La surface cylindrique $z = f(\alpha x + \beta y)$ satisfaisant à l'équation (11) est déterminée par l'équation

$$(A\alpha^2 + 2B\alpha\beta + C\beta^2) f'' = 0,$$

d'où l'on voit que *le théorème classique de Liouville peut devenir inapplicable à l'équation parabolique* (11) *dans le cas seulement, où les coefficients* $A(x, y)$, $B(x, y)$, $C(x, y)$ *sont proportionnels à des constantes convenables dans une bande infinie du plan* (x, y) *limitée pas deux droites parallèlles.*

On vérifie d'une façon analogue que *le théorème classique de Liouville est applicable à l'équation parabolique*

$$[a(x, y)\, p + b(x, y)\, q]^2 r + 2[a(x, y)\, p + b(x, y)\, q][b'(x, y)\, p + c(x, y)\, q] s + \\ + [b'(x, y)\, p + c(x, y)\, q]^2 t = 0, \tag{12}$$

où $a(x, y)$, $b(x, y)$, $b'(x, y)$, $c(x, y)$ *sont des fonctions continues dans tous les cas, à l'exception de celui où il existe deux constantes réelles* α, β *telles que l'on a*

$$a(x, y)\, \alpha^2 + (b(x, y) + b'(x, y))\, \alpha\beta + c(x, y)\, \beta^2 = 0$$

en tous les points de la bande $d_0 < \alpha x + \beta y < d_1$.

4. Théorème III. *Soit S une surface à croissance d'ordre inférieur à un qui satisfait à l'équation* (5) *et à la condition* (6) *ou* (7) *dans toutes les régions S_P de S, où l'on a* (4); *supposons de plus qu'il existe une constante $P_0 < P$ telle qu'en tous les points de la surface S où l'on aurait*

$$P_0 \leqslant p^2 + q^2 < P_0 + \varepsilon, \tag{13}$$

pour $\varepsilon > 0$ assez petit, on aurait nécessairement

$$AC - B^2 > 0. \tag{6}$$

Dans ces conditions la surface S est une surface cylindrique à génératrices horizontales, qui se réduit à un plan horizontal, si la surface contient effectivement des points satisfaisant à (13).

En effet, d'après ce qui précède, l'inégalité (6) entraîne

$$r = s = t = 0. \tag{9}$$

Donc, si la surface contient au moins un point, où l'on a (13), les égalités (9) se trouveront réalisées dans une certaine région contenant ce point de sorte que S se réduira à un plan

$$z = p_0 x + q_0 y + C \tag{14}$$

dans cette région. Par conséquent, dans ce cas toute la surface S est un plan, et l'on a nécessairement $p_0 = q_0 = P_0 = 0$. Au contraire, si la surface ne contient pas de points satisfaisant à (13), on aura en tous les points de la surface l'inégalité

$$p^2 + q^2 < P_0 < P \tag{15}$$

qui entraîne (6) ou (7) et par conséquent (10), d'où il résulte que S est une surface cylindrique à génératrices horizontales.

Nous voyons ainsi que les énoncés des théorèmes I et III qui se rapportent à des surfaces qui ne satisfont que partiellement à l'équation (5) excluent le cas où les coefficients A, B, C satisfont *identiquement* à l'égalité

$$AC - B^2 = 0. \tag{7}$$

Il est facile de montrer par un exemple que cette exception est inévitable, car le théorème de Liouville même *affaibli*, comme plus haut, *tombe en défaut dans ce cas*. En effet, définissons la surface S par l'équation

$$z = y + \frac{1}{6}(x^2 + y^2 - 1)^3 \tag{16}$$

à l'intérieur du cercle C, ayant pour équation $x^2 + y^2 = 1$, tandis qu'*à l'extérieur* de C la surface est définie par les équations

$$z = y \quad \text{pour} \quad |y| \leqslant 1;$$
$$z = \frac{3(|y|-1)+\frac{1}{|y|}}{y} \quad \text{pour} \quad |y| \geqslant 1; \tag{16 bis}$$

il résulte de (16 bis) que la surface S satisfait à l'équation

$$r = 0 \tag{17}$$

à l'extérieur du cercle C. Mais *à l'intérieur* de C on a, d'après (16)

$$p^2 + q^2 = 1 + 2y(x^2 + y^2 - 1)^2 + (x^2 + y^2)(x^2 + y^2 - 1)^4 \geqslant$$
$$\geqslant 1 - \frac{32\sqrt{5}}{125} + \frac{256}{3125} > \frac{1}{2};$$

par conséquent, *la surface S n'est pas cylindrique, quoiqu'elle est bornée et satisfait à l'équation* (17), *lorsque*,

$$p^2 + q^2 < \frac{1}{2}. \tag{4 bis}$$

5. Il est nécessaire d'observer que la condition que l'équation (5) soit du type *elliptique* (ou parabolique) n'est pas entièrement équivalente à (6) (ou (7)), lorsque A, B, C dépendent de r, s, t et la condition (6) (ou (7)) n'est pas remplie *identiquement*. Ainsi, l'équation de Monge — Ampère (sans second membre)

$$F = rt - s^2 + A_1 r + 2B_1 s + C_1 t = 0 \tag{18}$$

où A_1, B_1, C_1 sont des fonctions de x, y, z, p, q, qui est du type *elliptique* pour une solution donnée z, lorsqu'on a (après la substitution de z et de ses dérivées p, q)

$$A_1 C_1 - B_1^2 = F_r' F_t' - \frac{1}{4}(F_s')^2 > 0 \tag{19}$$

mise sous la forme (5), peut ne pas satisfaire à la condition (6). Il importe dans ces conditions de démontrer que

Le théorème classique de Liouville s'applique à toutes les solutions de l'équation (18) *de Monge — Ampère du type elliptique.*

A cet effet, démontrons d'abord le

Lemme. *Les surfaces satisfaisant à une équation* (18) *de Monge — Ampère du type elliptique* (19) *sont de deux espèces différentes: les surfaces de la première classe sont partout à courbure non positive* ($rt - s^2 \leqslant 0$); *les surfaces de la seconde classe sont partout à courbure positive* ($rt - s^2 \geqslant 4(A_1 C_1 - B_1^2) > 0$).

En effet, mettons l'équation (18) sous la forme (5):

$$F = \left(A_1 + \frac{t}{2}\right) r + 2\left(B_1 - \frac{s}{2}\right) s + \left(C_1 + \frac{r}{2}\right) t = 0;$$

on a donc

$$\left(A_1+\frac{t}{2}\right)rF=\left[A_1r+B_1s+\frac{1}{2}(rt-s^2)\right]^2+\left(A_1+\frac{t}{2}\right)\left(C_1+\frac{r}{2}\right)(rt-s^2)+$$
$$+\left[\left(A_1+\frac{t}{2}\right)\left(C_1+\frac{r}{2}\right)-\left(B_1-\frac{s}{2}\right)^2\right]s^2=0. \qquad (18\text{ bis})$$

Par conséquent, on a

$$H=\left(A_1+\frac{t}{2}\right)\left(C_1+\frac{r}{2}\right)-\left(B_1-\frac{s}{2}\right)^2>0 \qquad (20)$$

ou bien $H\leqslant 0$. Dans le cas, où c'est l'inégalité (20) qui a lieu, on a également $\left(A_1+\frac{t}{2}\right)\left(C_1+\frac{r}{2}\right)>0$; il résulte alors de (18 bis) que

$$rt-s^2\leqslant 0. \qquad (21)$$

Au contraire, si

$$H=A_1C_1-B_1^2+\frac{1}{2}(A_1r+2B_1s+C_1t)+\frac{1}{4}(rt-s^2)=$$
$$=A_1C_1-B_1^2-\frac{1}{4}(rt-s^2)\leqslant 0$$

on a

$$rt-s^2\geqslant 4(A_1C_1-B_1^2)>0, \qquad (22)$$

et le passage continu de (21) à (22) étant impossible, chaque solution de (18) appartient *entièrement* à la classe (21) ou à la classe (22).

Or, la croissance d'une surface de la classe (21) ne peut être d'ordre inférieur à un que lorsqu'elle se réduit à un plan horizontal, tandis que la croissance d'une surface satisfaisant à (22) n'est jamais d'ordre inférieur à un.

Corollaire. *Soient* $u_1(x, y)$, $u_2(x, y)$ *deux solutions de l'équation*

$$rt-s^2+A_1(x, y)r+2B_1(x, y)s+C_1(x, y)t=D(x, y) \qquad (23)$$

où

$$A_1C_1-B_1^2+D>0; \qquad (24)$$

si la croissance de $z=u_2-u_1$ *est d'ordre inférieur à un,* z *est égale à une constante.*

En effet, z satisfait à l'équation

$$\left(r+\frac{\partial^2u_1}{\partial x^2}\right)\left(t+\frac{\partial^2u_1}{\partial y^2}\right)-\left(s+\frac{\partial^2u_1}{\partial x\,\partial y}\right)^2+A_1\left(r+\frac{\partial^2u_1}{\partial x^2}\right)+2B_1\left(s+\frac{\partial^2u_1}{\partial x\,\partial y}\right)+$$
$$+C_1\left(t+\frac{\partial^2u_1}{\partial y^2}\right)=D(x, y),$$

c'est-à-dire à l'équation de Monge—Ampère sans second membre

$$rt-s^2+r\left(A_1+\frac{\partial^2u_1}{\partial y^2}\right)+2s\left(B_1-\frac{\partial^2u_1}{\partial x\,\partial y}\right)+t\left(C_1+\frac{\partial^2u_1}{\partial x^2}\right)=0$$

avec l'inégalité

$$\left(A_1+\frac{\partial^2u_1}{\partial y^2}\right)\left(C_1+\frac{\partial^2u_1}{\partial x^2}\right)-\left(B_1-\frac{\partial^2u_1}{\partial x\,\partial y}\right)^2=A_1C_1-B_1^2+D>0.$$

Le dernier corollaire ne s'étend pas, évidemment, aux équations paraboliques.

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LITTÉRATURE CITÉE

[1] S. Bernstein, Bull. Acad. Sci. URSS, sér. math. (1942).

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MATHÉMATIQUES

APPROXIMATION PAR POLYNÔMES DES FONCTIONS VÉRIFIANT LA CONDITION DE LIPSCHITZ

Par S. NIKOLSKY

(Présenté par A. N. Kolmogoroff, de l'Académie, le 31. VIII. 1943)

Soit MH la classe des fonctions $f(x)$ définies dans l'intervalle $(-1, +1)$ et vérifiant dans cet intervalle la condition de Lipschitz avec la constante M. Quels que soient x' et x'' appartenant à $(-1, +1)$, on a l'inégalité

$$|f(x') - f(x'')| \leqslant M|x' - x''|.$$

Désignons par

$$P_k(x) = \cos k \arccos x \qquad (k = 0, 1, 2, \ldots)$$

les polynômes de Tschebycheff et par

$$a_k = \frac{2}{\pi} \int_{-1}^{+1} \frac{f(u) P_k(u)}{\sqrt{1-u^2}}\, du \qquad (k = 0, 1, 2, \ldots)$$

les coefficients de Fourier de $f(x)$ par rapport aux polynômes $P_k(x)$ avec le poids $(1-x^2)^{-1/2}$.

A chaque système $\{\eta\}$ de nombres $\eta_1, \eta_2, \ldots, \eta_{n-1}$ il correspond un polynôme de degré $n-1$

$$U_n(\eta; f, x) = \frac{a_0}{2} + \sum_{k=1}^{n-1} \eta_k a_k P_k(x) \tag{1}$$

que nous considérerons comme une approximation de la fonction $f(x)$ (au point x).

La borne supérieure

$$E_n(\eta; MH) = \sup_{f \in MH} \sup_{-1 \leqslant x \leqslant 1} |f(x) - U_n(\eta; f, x)| \tag{2}$$

des écarts des fonctions $f(x)$ de leurs approximations, étendue à toute la classe MH des fonctions et à l'intervalle $(-1, +1)$, caractérise la mesure de l'approximation de la classe MH au moyen des polynômes (1). C'est pourquoi il est intéressant de résoudre le problème suivant: trouver le système des nombres $\{\eta\}$ pour lequel cette borne supérieure (2) est la plus petite, n étant donné. Le théorème suivant donne une solution asymptotique de ce problème.

Théorème. *Quel que soit le système $\{\eta\}$ des nombres $\eta_1, \eta_2, \ldots, \eta_{n-1}$, on a l'inégalité*

$$E_n(\eta; MH) \geqslant \frac{M\pi}{2n} + \varepsilon_n, \tag{3}$$

où ε_n ne dépend pas des nombres γ_{lk}, mais dépend seulement de n et il est d'ordre

$$\varepsilon_n = O\left(\frac{1}{n^2}\right). \tag{4}$$

D'autre part, pour le système $\{\eta_l^{(n)}\}$ des nombres

$$\eta_k^{(n)} = \frac{k\pi}{2n} \operatorname{ctg} \frac{k\pi}{2n} \qquad (k = 1, 2, \ldots, n-1) \tag{5}$$

on a

$$E_n(\eta^{(n)}; MH) < \frac{M\pi}{2n}. \tag{6}$$

Démonstration. Faisons la substitution $x = \cos\theta\ (0 \leqslant \theta \leqslant \pi)$. La fonction $f(\cos\theta)$ que nous considérerons maintenant sur tout l'axe réel est paire, de période 2π et absolument continue, si $f \in MH$. Donc, en intégrant par parties sa série de Fourier, nous aurons

$$f(\cos\theta) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{1}{\pi} \int_0^{2\pi} \cos k(t-\theta) f(\cos t)\, dt =$$

$$= \frac{a_0}{2} + \frac{1}{\pi} \int_0^{2\pi} \psi(t) \sin(t+\theta) f'[\cos(t+\theta)]\, dt$$

où

$$\psi(t) = \sum_{k=1}^{\infty} \frac{\sin kt}{k}. \tag{7}$$

D'une manière analogue, en tenant compte de ce que $f(\cos\theta)$ est paire, on aura

$$U_n(\gamma_l; f, \theta) = \frac{a_0}{2} + \sum_{k=1}^{n-1} \gamma_{lk} \frac{1}{\pi} \int_0^{2\pi} \cos k(t-\theta) f(\cos t)\, dt =$$

$$= \frac{a_0}{2} + \frac{1}{\pi} \sum_{k=1}^{n-1} \frac{\gamma_{lk}}{k} \int_0^{2\pi} \sin kt \sin(t+\theta) f'[\cos(t+\theta)]\, dt.$$

Ainsi

$$|f(\cos\theta) - U_n(\gamma_l; f, \theta)| = \frac{1}{\pi} \int_0^{2\pi} \left(\psi(t) - \sum_{k=1}^{n-1} \frac{\gamma_{lk}}{k} \sin kt\right) \sin(t+\theta) f'[\cos(t+\theta)]\, dt \Big| \tag{8}$$

Considérons dans l'intervalle $(-1, +1)$ la fonction continue $f_n(x)$ dont la dérivée est égale à

$$f_n'(x) = M \operatorname{sign} \cos(n+1) \arccos x.$$

Il est évident que $f_n \in MH$. Posons $\theta_* = \frac{\pi}{2}$, si n est pair, et $\theta_* = \frac{n\pi}{2(n+1)}$, si n est impair. Dans les deux cas

$$f_n'[\cos(t+\theta_*)] = \pm M \operatorname{sign} \sin(n+1)t = \pm \frac{4M}{\pi} \sum_{v=0}^{\infty} \frac{\sin(2v+1)(n+1)t}{2v+1}.$$

Il suit donc de (7) et (8) pour un système arbitraire $\{\gamma_l\}$

$$f_n(\cos\theta_*) - U_n(\gamma_l; f_n, \theta_*)| = \frac{4M}{\pi^2} \sin\theta_* \int_0^{2\pi} \left(\sum_{k=1}^{\infty} \frac{\sin kt \cos t}{k}\right) \left(\sum_{v=0}^{\infty} \frac{\sin(2v+1)(n+1)t}{2v+1}\right) dt =$$

$$= \frac{4M}{\pi^2} \sin\theta_* \int_0^{2\pi} \left(\sum_{k=1}^{\infty} \frac{\sin(k+1)t + \sin(k-1)t}{2k} \right) \left(\sum_{v=0}^{\infty} \frac{\sin(2v+1)(n+1)t}{2v+1} \right) dt =$$

$$= \frac{4M}{\pi^2} \sin\theta_* \int_0^{\pi} \sum_{v=0}^{\infty} \frac{\sin^2(2v+1)(n+1)t}{2v+1} \left(\frac{1}{(2v+1)(n+1)-1} + \frac{1}{(2v+1)(n+1)+1} \right) dt =$$

$$= \frac{4M}{\pi} \sin\theta_* \sum_{v=0}^{\infty} \frac{n+1}{(2v+1)^2(n+1)^2-1} = \frac{4M}{\pi} \frac{1}{n} \sum_{v=0}^{\infty} \frac{1}{(2v+1)^2} + \varepsilon_n = \frac{M\pi}{2n} + \varepsilon_n,$$

où ε_n vérifie évidemment l'égalité (1) et ne dépend pas des nombres η_k. Et comme la différence dans le premier membre de la chaîne des égalités écrites ne surpasse pas (2), l'inégalité (3) est ainsi démontrée.

Pour obtenir l'inégalité (6) nous profiterons d'une méthode utilisée par Favard [1] pour résoudre un problème analogue pour la classe des fonctions qui possèdent dans l'intervalle $(-1, +1)$ une dérivée dont la valeur absolue ne surpasse pas $(1-x^2)^{-\frac{1}{2}}$.

A savoir, les nombres $\eta_k^{(n)}$ sont choisis de telle manière que la différence

$$\psi(t) - \sum_{k=1}^{n-1} \frac{\eta_k^{(n)}}{k} \sin kt \tag{9}$$

s'annulle aux points $t_k = \frac{k\pi}{n}$ $(k = 1, 2, \ldots, n-1)$. Ainsi qu'il a été démontré par Favard [voir aussi [2]], dans ce cas la différence (9) change dans l'intervalle $0 < t < \pi$ son signe en ces points et ces points seulement. Il est facile à calculer que les nombres $\eta_k^{(n)}$ ainsi définis s'expriment par la formule (5).

Maintenant, si $f \in MH$, en vertu de (8) on a pour chaque θ l'inégalité

$$f(\cos\theta) - U_n(\eta^{(n)}; f, \theta) \leqslant \frac{M}{\pi} \int_0^{2\pi} \left| \psi(t) - \sum_{k=1}^{n-1} \frac{\eta_k^{(n)}}{k} \sin kt \right| |\sin(t+\theta)| \, dt = F_n(\theta).$$

Posons $\max F_n(\theta) = F_n$. Alors on a évidemment

$$E_n(\eta^{(n)}, MH) \leqslant F_n < \frac{2M}{\pi} \int_0^{\pi} \left| \psi(t) - \sum_{k=1}^{n-1} \frac{\eta_k^{(n)}}{k} \sin kt \right| dt =$$

$$= \frac{2M}{\pi} \left| \int_0^{\pi} \left(\psi(t) - \sum_{k=1}^{n-1} \frac{\eta_k^{(n)}}{k} \sin kt \right) \operatorname{sign} \sin nt \, dt \right| = \frac{M\pi}{n}$$

et (6) est ainsi démontré.

Remarque. 1. La méthode d'approximation $U_n(\eta^{(n)}; f, x)$, où les $\eta_k^{(n)}$ sont définis par les égalités (5), étant asymptotiquement la meilleure pour la classe MH parmi toutes les méthodes $U_n(\eta; f, x)$ correspondant aux systèmes $\{\eta\}$ différents, tend uniformément vers $f(x)$ pour chaque fonction f continue dans l'intervalle $(-1, +1)$.

En effet, la substitution $x = \cos\theta$ donne

$$U_n(\eta^{(n)}; f, \theta) = \frac{a_0}{2} + \sum_{k=1}^{n-1} \eta_k^{(n)} a_k \cos k\theta,$$

où a_k sont les coefficients de Fourier de la fonction $f(\cos\theta)$, et le système des nombres $\eta_0^{(n)}, \eta_1^{(n)}, \ldots, \eta_n^{(n)}$, pour lequel

$$\eta_0^{(n)} = 1, \qquad \eta_n^{(n)} = 0, \qquad (n = 1, 2, \ldots)$$

vérifie la condition de convexité

$$\Delta_k^2 = \Delta_k - \Delta_{k+1} \leqslant 0, \qquad \Delta_k = \lambda_k^{(n)} - \lambda_{k+1}^{(n)}, \qquad [k = 0, 1, \ldots, (n+1)]$$

ce qui résulte de ce que la fonction $x \operatorname{ctg} x$ a la seconde dérivée négative pour $0 < x < \frac{\pi}{2}$. D'ailleurs, les conditions suivantes sont vérifiées:

$$1)\ \eta_k^{(n)} \to 1 \text{ pour } n \to \infty;$$

$$2)\ \sum_{k=1}^{n-1} \frac{\eta_{n-k}^{(n)}}{k} = \sum_{k=1}^{n-1} \frac{1}{k} \frac{(n-k)\pi}{2n} \operatorname{ctg} \frac{(n-k)\pi}{2n} < \frac{\pi}{2} \sum_{k=1}^{n-1} \frac{1}{k} \cos \frac{(n-k)\pi}{2n} =$$

$$= \frac{\pi}{2} \sum_{k=1}^{n-1} \frac{1}{k} \sin \frac{k\pi}{2n} < \frac{\pi^2}{4} \frac{n-1}{n} < C,$$

où C est une constante absolue.

Ces propriétés entraînent, ainsi que j'ai démontré, l'égalité

$$\lim_{n \to \infty} U_n(\eta^{(n)}; f, \theta) = f(\cos\theta),$$

la convergence étant uniforme pour chaque fonction continue.

Remarque 2. Posant

$$E_n(\eta; MH, x) = \sup_{f \in MH} |f(x) - U_n(\eta; f, x)|,$$

nous avons les évolutions suivantes dont la première précise l'inégalité (6)

$$E_n(\eta^{(n)}; MH, x) \leqslant M \frac{\pi}{2} \frac{\sqrt{1-x^2}}{n} + M\,|x|\, O\left(\frac{\lg n}{n^2}\right) \tag{10}$$

$$E_n(1; MH, x) = \frac{4}{\pi^2} \frac{\lg n}{n} \sqrt{1-x^2} + O\left(\frac{1}{n}\right) \tag{11}$$

$$\eta_{1,k} = 1 \qquad (k = 1, 2, \ldots, n-1)$$

uniformément par rapport à x dans l'intervalle $(-1 \leqslant x \leqslant 1)$.

Pour $x = 0$ la partie droite de (10) est distincte de la partie gauche par $O\left(\frac{1}{n^2}\right)$. Pour $x = 1$ l'inégalité (10) se transforme en une égalité dans ce sens que O ne peut être remplacé par o.

L'égalité asymptotique (11) montre que la fonction f étant développée ordinairement en une série de Fourier par rapport aux polynômes de Tschebycheff, l'approximation de ses sommes partielles à la fonction est plus grossière que celle obtenue au moyen des coefficients de $\{\eta_i^{(n)}\}$.

La démonstration sera publiée dans un autre recueil.

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ASTRONOMY

CONNEXION BETWEEN FACULAE AND GEOMAGNETIC ACTIVITY

By E. R. MUSTEL

(Communicated by G. A. Shajn, Member of the Academy, 24.X.1943)

Early in 1942, at the Sternberg State Astronomical Institute there were started regular observations of solar spots and faculae. A comparison of data characterizing the magnetic field of the Earth daily obtained from the Kosulin Geomagnetic Observatory (Wysokaya Dubrava) with the details observed at the Sun surface enabled the author in February of the same year to reveal the following relation: every facular field passing through the apparent centre of the disk of the Sun was found to be accompanied by disturbances of a certain strength which may sometimes be rather considerable. We at once made use of this experimental fact as one of the principal indices for forecasting geomagnetic and ionospheric disturbances.

The using of facular fields as a source of geomagnetic disturbances made it possible in some cases to forecast the latter with an accuracy of up to 24 hours, and occasionally even up to 12 hours. Along with this great attention was given to further experimental substantiation of the results obtained. This proceeded along two lines.

a) Checking up of those geomagnetic forecasts that were based on the passing of facular fields through the apparent centre of the Sun disk. In particular, at the beginning of 1943 comparison was made of all the geomagnetic disturbances which had occurred in 1942, with the corresponding details observed upon the Sun surface, especially with the focular field. This comparison enabled the author to arrive at the following conclusions:

1) About 70 per cent of the thirty geomagnetic disturbances examined are obviously related with those facular fields which were passing through the apparent centre of the disk of the Sun. In most of the cases the geomagnetic disturbances set in one—two days after the corresponding facular field had passed through the central meridian of the Sun. In this connexion it should be pointed out that the comparison made between the facular and the geomagnetic disturbances in 1942 was greatly facilitated by the fact that, on the one hand, geomagnetic activity was very intense during that year, and on the other hand, by that Sun spots which might interfere with the uniformness in the case of such a comparison, were rather few.

Obviously enough, the reliability of the results obtained by us is materially dependent on this circumstance.

2) Very weak, though stable, facular regions, i. e. such as have existed for a long period, may sometimes produce very important disturbances. Here by using the term of f a c u l a r r e g i o n s we do not necessarily understand those regions which are wholly filled by facular formations. On the contrary,

as geoactive facular regions such regions may be considered which are occasionally occupied by very small light facular spots and points; sometimes liable to rapidly change their appearance and intensity. The position of these formations within the boundaries of the region considered is also subject to considerable variation.

The existence of such stable facular regions (in the sense of their longevity) is a well-known fact. In connexion with the data obtained, the lifetime of some of the facular regions which were observed in 1942 and recurred at each revolution of the Sun, goes fully to account for the 27-day recurrence tendency in magnetic conditions, which was rather sharply expressed in 1942*.

And it could by no means be interpreted in this case from the viewpoint of the Sun spot activity.

3) A single facular region may produce disturbances stretching over a much larger day-interval than the interval occupied by this region. We seem to deal here with a high dispersion of velocities in particles ejected by the faculae. Any factor of prolonged after-effect is from a purely physical standpoint scarcely likely to occur in such regions as are referrable to the envelope surrounding the Earth.

4) In the course of 1942 such cases were also recorded (no more than 3—4 in number) where facular fields passing through the apparent centre of the Sun disk, failed to produce geomagnetic disturbances. In all those cases, the corresponding fields were invariably found to comprise Sun spots. This rule did not hold for all cases; however. Namely; a whole array of facular regions passing through the apparent centre of the disk of the Sun and inclosing spots were found to produce magnetic disturbances of various strength.

In this connexion the Sun spots may be considered as a certain disturbing factor**.

b) Checking up on the basis of observational material available in literature.

To this purpose use was made of records taken within the interval 1929—1935, as one including the phase of solar activity corresponding to that in 1942. The investigation involved the Meudon synoptic diagrams (in the part of flocculi), the Zürich synoptic diagrams (facular fields), and; lastly; the catalogue of Sun spot activity for 1932—1937 published at Pulkovo (1). As a source of magnetic data use was made of the observations of the Stonyhurst Observatory, presented graphically in the form of 27-day recurrence tendency diagrams. A comparison of geomagnetic disturbances with the facular and floccular fields referred to in the records listed above, go fully to confirm the results obtained at Sverdlovsk.

Thus, for example; in 1930 out of the total of 26 more or less important disturbances, about 21, i. e. 81 per cent, may in some way be found to coincide with the passing of facular and floccular fields through the apparent centre of the disk of the Sun; in 1931 the same should be said of 22 disturbances out of 31, i. e. of 71 per cent. It stands to reason that the data obtained hold true and remain efficient only for the 4—5 year period that preceded the minimum of Sun spot activity.

After this period the solar formations considered here pass to high heliographic latitudes; and such a comparison becomes meaningless. For moment next to that of the maximum of Sun spot activity, as well as during 1—2 years following this moment, the above referred relation between the geomagnetic disturbances and the faculae cannot be possibly revealed for the same reason. Unlike this, for the phase of Sun spot activity comprising, say, year 1942, this relation is clearly expressed, which is evidenced by the observations carried out at Sverdlovsk.

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* Of course, we speak here of those regions which involved the apparent centre of the Sun disk.

** The nature of this factor will be discussed below.

The correspondence between the facular fields passing through the apparent centre of the Sun disk and the geomagnetic disturbances which they produce; as well as the lack of such a coincidence in nearly all cases where the facular fields do not involve the centre of the Sun disk; seem to be due to faculae ejecting geoactive particles in a radial direction, i. e. in a direction normal to the surface of the Sun at a given place.

It is of interest to point our here the results obtained upon the treatment of the coronal plates taken by the Soviet expeditions at the total solar eclipse of 1936.

The treatment has revealed (2) a radial outflow of coronal matter in the equatorial regions above the flocculae in the shape of powerful streamers. This points to an intimate connexion between the coronal phenomena as obviously non-static ones; and the phenomena of ejection of geomagnetic corpuscles.

Aside from the 27-day periodicity, the results secured by us do fully account for the seasonal character of the geomagnetic activity, clearly expressed in 1942 and related with the change in the heliographic latitude of the apparent centre of the Sun disk.

It should be noted that for 1942 Sun spot activity gives nothing again in this respect.

From all that has been said here it follows that faculae, and consequently flocculae, as well, should be regarded as one of the principal sources of geoactive corpuscular radiation. It would be appropriate to cite here the results secured by Goodall (3), as closely related to what has been discussed here. This writer has found for the period 1933—1938 a close relation to exist between the critical frequencies of the region F_2 and the average international values characterizing the floccular activity in the central zone of the Sun disk. This relation is much closer than the one we find when comparing the critical frequencies with other indices of Sun spot activity. The graph adjoined by Goodall to his note is a convincing proof of this fact.

We are thus brought to assume that the disturbances that are usually being associated with the Sun spots are as a matter of fact connected with the facular (floccular) fields rather than with the spots proper. The rôle of the spots as of formations characterized by considerable magnetic fields is reduced to merely deflecting the charged corpuscles ejected directly from the faculae; on account of which the radial motion of those corpuscles may be disturbed. The above-cited data as to the facular fields which have passed through the apparent centre of the solar disk, without, however, producing geomagnetic disturbances, because of the spots present in them, speak in favour of this interpretation. Magnetic disturbances, inclusive also of great magnetic storms, due to the passing of Sun spots through the central meridian of the Sun, but not through the centre of the disk, should then also be associated with the deflection of the corpuscles ejected by the faculae surrounding these spots. Indeed, very powerful facular fields, though deprived of Sun spots and passing through the central meridian without involving the centre of the Sun disk, are as a rule not accompanied by disturbances.

Exclusions to this rule may, however, also be readily explained from the standpoint of the so-called «invisible spots» which are not infrequently present in the flocculae; and occasionally rise to the surface. Although the magnetic field of those spots is not very large (no more than 500 Gauss), nevertheless it is quite sufficient to deflect appreciably the radially moving charged particles.

Thus, from the standpoint of the considerations here developed it should be assumed that all the geomagnetic disturbances of a purely corpuscular nature are due to the deflecting action of the magnetic field of the Sun spots. It stands to reason that we do not speak here of such disturbances as are due to the passing of the facular regions through the apparent centre of the Sun disk. And the source of the corresponding corpuscles are the facular (floccular) regions.

This conclusion is especially true of that phase of the Sun spot activity at which all the faculae and spots are situated at high heliographic latitudes. This

statement should be qualified in the sense that only disturbances of corpuscular nature are meant here. We will not speak here of disturbances connected with the emission by the Sun of ultra-violet radiation of a strongly increased intensity, let us only point out that this factor is in this respect of particular importance on account of a number of considerations.

Naturally enough, our considerations concerning the facular fields as the chief sources of corpuscular radiation stand in need of further discussion. As to the relation between the passing of facular fields through the apparent centre of the solar disk and the geomagnetic disturbances, discussed in the first part of this article, the question seems to be clear enough. The importance of the relation found consists in that it may be regarded as a rule bearing no pronounced statistical character, unlike that in the case of the passing of a group of Sun spots through the central meridian of the Sun.

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HYDROMECHANICS

DECAY OF ISOTROPIC TURBULENT MOTIONS OF INCOMPRESSIBLE FLUID

By L. I. SEDOV

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In the case of isotropic turbulent motion of an incompressible fluid it follows from the conditions of symmetry that the determination of the correlation tensors for the second and third order velocities reduces to finding two functions (1)

$$b_d^d = b(t) f(r,t) = \overline{v_d(M,t)\, v_d(M_1,t)}$$

$$b_d^{nn} = b^2 h(r,t) = \overline{v_d(M,t)\, v_n(M_1,t)\, v_n(M_1,t)}$$

where t is time; r, distance from point M to point M_1; v_d and v_n are the projections of the instantaneous velocity on to the intercept MM_1 and on to a direction perpendicular to it, respectively. The line overhead denotes averaging. Function $b(t)$ is defined by the equation:

$$b(t) = b_d^d(0,t).$$

For the functions b_d^d and b_d^{nn} the equation of Navier-Stokes gives only one relation (1,2)

$$\nu\left(\frac{\partial^2 b_d^d}{\partial r^2} + \frac{4}{r}\frac{\partial b_d^d}{\partial r}\right) - \frac{1}{2}\frac{\partial b_d^{\alpha}}{\partial t} = \frac{\partial b_d^{nn}}{\partial r} + \frac{4}{r} b_d^{nn} \qquad (1)$$

where ν is the coefficient of kinematic viscosity.

In the closing stage of motion, when the instantaneous velocities are tending to zero, the quantity b_d^{nn} may be neglected in comparison with b_d^d; this case has been discussed by Kármán and Howarth (1) who have assumed in addition that the function f depends solely upon the combination $\frac{r^2}{\nu t}$.

It may be easily realized that the latter assumption amounts to supposing the asymptotic law of the behaviour of b_d^d to have no other connexion with the initial perturbations but through a certain multiplicative constant A having the dimensionality $L^p T^q$ with arbitrary p and q. On this assumption we have from the theory of dimensions

$$b_d^d = \frac{A\nu^{1-\frac{p}{2}}}{t^{10\alpha}} f(\xi); \qquad \left(b = \frac{A\nu^{1-\frac{p}{2}}}{t^{10\alpha}}\right) \qquad (2)$$

where $\xi = \frac{r^2}{\nu t}$ and $\alpha = 1 + q + \frac{p}{2}$.

Putting $b_d^{nn} = 0$ and making use of formula (2), we obtain from equation (1)

$$f'' + \left(\frac{1}{8} + \frac{3}{2\xi}\right) f' + \frac{5\alpha}{\xi} f = 0. \tag{3}$$

The solution of equation (3) for $f(0) = 1$ takes the form (3)

$$f(\xi) = M\left(10\alpha, \frac{5}{2}, -\frac{\xi}{8}\right)$$

where $M(\alpha, \gamma, x)$ is a confluent hypergeometric function. For any $\alpha > 0$ we have $f(\infty) = 0$; therefore for $r \neq 0$ and $t = 0$ we have $f = 0$; for $r = 0$ and t arbitrary, $f = 1$. For determining the constant α more data are necessary.

The asymptotic behaviour of the moment b_d^d for $r \to +\infty$ or for $t \to 0$ is easily revealed by means of the relation

$$\lim_{\frac{r^2}{vt} \to \infty} b_d^d r^{10\alpha} = Av^{2+q} \frac{\Gamma\left(\frac{5}{2}\right) 8^{10\alpha}}{\Gamma\left(\frac{5}{2} - 10\alpha\right)}. \tag{5}$$

If $\alpha = 1/4 + k \cdot 0.1$; where k is a positive integer, then a simple formula is valid for $f(\xi)$:

$$f(\xi) = \frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(k + \frac{5}{2}\right)} \left[\frac{d^k}{dt^k}\left(t^{k+\frac{3}{2}} e^{-\frac{\xi t}{8}}\right)\right]_{t=1} \tag{6}$$

In that case the distribution of values b_d^d for $t = 0$ bears the character of a source; for $r = 0$ we have $b_d^d = \infty$; and for $r \neq 0$ we have $b_d^d = 0$.

The case of a source of perturbations was discussed by Loytzansky (2) and Millionshtchikov (4) who arrived at the conclusion that Kárman's constant α is equal to 1/4. It should be noted, however, that this conclusion was not reached without some additional assumptions.

As has been pointed out by Loytzansky (4); it follows from equation (1) that the quantity Λ defined by the formula

$$\Lambda = \int_0^\infty b_d^d r^4 dr \tag{7}$$

does not depend upon time.

It may be easily shown that having a solution in the form (2) we have $\Lambda = 0$ when $\alpha > 1/4$; $\Lambda = \infty$ when $\alpha < 1/4$; and only for $\alpha = 1/4$ is the invariant Λ finite and distinct from zero.

In investigating the damping laws when the pulsations are large enough, moments of the third order have to be taken into consideration. In order to obtain the laws of the decay of turbulence with allowance for moments of the third order, let us adopt the assumption of Kárman and Howarth which takes the form

$$b_d^d = bf(\chi) \tag{8}$$

$$b_d^{nn} = b^{\frac{3}{2}} h(\chi) \tag{9}$$

where $\chi = \frac{r}{l}$ and l is a linear quantity which may depend upon time and characteristic geometrical and physical constants.

In virtue of formulae (8) and (9) the equation (1) gives

$$h'(\chi) + \frac{4h(\chi)}{\chi} - \frac{1}{2}\chi f'(\chi) \frac{1}{\sqrt{b}} \frac{dl}{dt} + \frac{1}{2} f(\chi) \frac{l}{\sqrt{b^3}} \frac{db}{dt} = \frac{\nu}{\sqrt{b} \cdot l}\left(f''(\chi) + \frac{4f'(\chi)}{\chi}\right) \tag{10}$$

Movements with larger pulsations are characterized by larger values of the parameter $\frac{\sqrt{b}\cdot l}{\nu}$. On this consideration Kárman and Howarth (1) replaced the right-hand side of equation (10) by zero; after which the following formulae may be derived:

$$\frac{l}{l_0}=\left(\frac{b_0}{b}\right)^k \tag{11}$$

and

$$\frac{b}{b_0}=\left(\frac{t+t_0}{2t_0}\right)^{-\frac{2}{2k+1}};\quad \frac{l}{l_0}=\left(\frac{t+t_0}{2t_0}\right)^{\frac{2k}{2k+1}} \tag{12}$$

where $k>-1/2$ is an arbitrary constant. By means of the invariant Λ A. N. Kolmogoroff (5) has obtained the values of the exponents in formulae (12).

In fact, we have

$$\Lambda=b\int_0^\infty f\left(\frac{r}{l}\right)r^4dr=bl^5\int_0^\infty f(\chi)\,\chi^4d\chi \tag{13}$$

Comparing the formulae (11) and (13) we get $k=1/5$. Apparently, this conclusion is based on the assumption that the invariant Λ is finite and distinct from zero.

In the approximate solution by Kárman and Howarth no relations are given for determining the functions $f(\chi)$ and $h(\chi)$.

Being given the fundamental conditions that $f(0)=1$, $f(\infty)=0$ and $h(0)=h'(0)=h''(0)=0$ (1), I have obtained rigorous solutions of equation (10) by which not merely the shape of the functions $b(t)$ and $l(t)$, but also the functions $f(\chi)$ and $h(\chi)$ are determined. If the particular case of $k=1/2$, when the solution presented by the formulae (12) is an exact solution, be omitted from consideration, we shall find that the unique solution of equation (10) satisfying the above-mentioned conditions has the form

$$f(\chi)=M\left(10\alpha,\frac{5}{2},-\frac{\chi^2}{8}\right) \tag{14}$$

$$h(\chi)=\frac{p}{2}\cdot\frac{1}{\chi^4}\int_0^\chi \chi^5 f'(\chi)\,d\chi \tag{15}$$

and for l and b we have the differential equations

$$\frac{1}{\sqrt{b}}\frac{dl}{dt}=\frac{1}{2}\frac{\nu}{\sqrt{b}\cdot l}+p \tag{16}$$

$$\frac{l}{\sqrt{b^3}}\frac{db}{dt}=-10\alpha\frac{\nu}{\sqrt{b}\cdot l} \tag{17}$$

where p and $\alpha>0$ are arbitrary constants.

For the quantity $f=\frac{b_a^2}{b}$ the formulae (4) and (14) give equal values if we set

$$\xi=\chi^2=\frac{r^2}{l^2}.$$

In particular, if $p=0$, the equations (16) and (17) give

$$l=\sqrt{\nu t};\quad b=\frac{A'}{(\nu t)^{10\alpha}}$$

and we have, moreover,

$$h=0 \text{ and } f=M\left(10\alpha, \frac{5}{2}, -\frac{\xi}{8}\right).$$

The result is a solution coinciding with that of Kárman and Howarth for small pulsations.

With $\alpha=1/4$ the invariant Λ is finite and different from zero, but then it follows from the relations (13), (16) and (17) that $p=0$ in this case. Hence for $\alpha=1/4$ the moments of order three are zero.

Thus, for $h\neq 0$ it may be inferred from the assumptions (8) and (9) that $\alpha\neq 1/4$; and the invariant Λ is either zero or infinitely great.

The general solution of equations (16) and (17) for $p\neq 0$ and $\alpha\neq 0.1$ is conveniently represented in the form

$$\frac{l}{l^*}=\frac{1}{w^{\frac{1}{20\alpha}}+\sqrt{w}} \tag{18}$$

$$\frac{\nu(t+t^*)}{l^{*2}}=\frac{1}{10\alpha}\int_w^\infty \frac{dw}{w^{1+\frac{1}{10\alpha}}\left(1+w^{\frac{10\alpha-1}{20\alpha}}\right)^2} \tag{19}$$

where $w=\frac{b}{b^*}$, $l^*=\frac{(10\alpha-1)\nu}{2p\sqrt{b^*}}$, b^* and t^* denoting the constants of integration.

The laws (18) and (19), bringing out the dependence of the average values of the squared projection of velocity b and the linear scale l upon time, contain only one essential arbitrary constant α, for b^* and l^* play the part of scale constants, and the constant t^* depends solely upon the choice of the point from which time is measured.

With $t\to+\infty$ the damping laws (18) and (19) derived here tend for any $\alpha>0.1$ asymptotically to the damping laws found by Kárman and Howarth while neglecting the moments of the third order.

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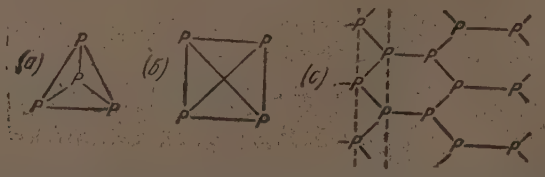
CHEMISTRY

STRUCTURE AND PROPERTIES OF THE MODIFICATIONS OF PHOSPHORUS AS A CONSEQUENCE OF ITS ATOMIC STRUCTURE

By J. V. KHODAKOV

(Communicated by V. G. Chlopin, Member of the Academy, 28. VIII.1943)

Since in the outer shell of the atom of phosphorus three electrons are deficient, this element is capable of forming three co-valent linkages (1) which, judging from the structures of PF_3; PCl_3; PI_3, $P(CH_3)_3$, etc.(2); show a tendency to be arranged with respect to each other at an angle of about 100°. At the expense of these linkages an atom of phosphorus (P^1) may combine with three others (P^2; P^3 and P^4). But the latter have two more linkages each, which they may spend either to interlink with each other whereby a closed-chain molecule P_4 with a structure formula *a* or *b* (white phosphorus) is formed; or to combine with further atoms of phosphorus, and then instead of closed-chain molecules; infinitely



extended (coordination) lattices arise corresponding to non-volatile modifications of phosphorus with the general structure formula *c*. These modifications must, moreover, be infusible and soluble in no solvent.

Since the atoms in the molecule P_4 are equivalent and each of them is directly connected with the three remaining ones, the space configuration of the molecule has to correspond the following conditions:

1) The atoms must occupy geometrically equivalent positions.

2) Any three atoms must occupy geometrically equivalent positions in relation to the fourth one.

The former condition is fulfilled in two configurations (besides the intermediate forms): square and tetrahedron; but only the latter fulfills at the same time the second condition: It is easily seen that the two structure formulae of P_4 advanced above are different projections of tetrahedral molecule upon the plane of the drawing.

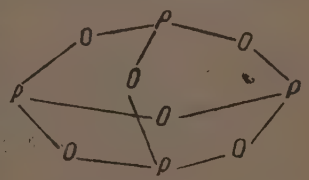
A disadvantage with the tetrahedral structure is that, instead of natural valence angles, it forces the atoms of phosphorus to assume the much narrower angle of 60°. Here, probably, lies the cause for metastability of white phosphorus*.

* On account of strains, analogous with Bayer's strains in lower polymethylenes.

Created in a «forcible way» (under the conditions of thermal decomposition, desaggregation of non-volatile modifications); this form tends through intracrystalline rearrangement of linkages to turn back from the molecular into the coordination lattice with valent angles nearer to their natural value.

The tetrahedral structure of the molecule of phosphorus in the vapour state, with the edge of the tetrahedron being 2.21 Å, has been established by the electronographic investigation of Maxwell, Mosley and Hedricks (3), and the structure of one of non-volatile modifications—black phosphorus—was ascertained by Gingrich, Hultgren and Warren (4). They found a coordination lattice wherein every atom has three neighbours, as well as in P_4 molecule, which are removed to almost the same distance as in P_4, namely 2.17, 2.17 and 2.20 Å, but have larger angles between the bonds, i. e. 99, 99 and 103°30′ (as compared with 60° in P_4)*. According to these authors, commercial red phosphorus has analogous, though less regular structure.

Now let the covalent bonds of phosphorus be used up in combining with oxygen atoms. Upon the condition that each of the three oxygen atoms coordinated with any atom of phosphorus be distributed between three different atoms of phosphorus, the simplest structure would be as follows (see right).



It is a regular octahedron formed of oxygen atoms. The centres of four out of eight faces should be occupied by atoms of phosphorus; but the demand for maintaining the valence angles compels these atoms of phosphorus to be displaced along the normals of these faces, and thus to become vertices of obtuse pyramids resting thereon. Actualy that very structure has been established for phosphorous anhydride (5); the molecular lattice in this case being stable, since the valence angles are close to the natural ones:

$$\angle \mathrm{OPO} = 99^\circ, \quad \angle \mathrm{POP} = 127^\circ \quad [\text{and in } (C_6H_4I)_2O \ \angle \mathrm{COC} = 118^\circ (6)].$$

The structure of P_4O_6 may also be described otherwise. It is the same tetrahedron but somewhat enlarged in dimensions in comparison with the P_4 molecule (the edge being 2.95 Å instead of 2.21 Å); and having the oxygen atoms attached to it. This structural comparison opens the way to a descriptive geometrical interpretation of the chain theory of oxidation of phosphorus vapour, as evolved by N. N. Semenov.

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* The lattice is laminated and corresponds with the scheme *c*; normalization of the angles is attained at the expense of ruffling the planar scheme *c* along the dotted lines.

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CHEMISTRY

SEEDS OF *THERMOPSIS LANCEOLATA* R. Br. AS A SOURCE OF CYTISINE

By M. V. TSAREV

(Communicated by V. G. Chlopin, Member of the Academy, 26.VIII.1943)

The alkaloid cytisine is recognized as a stronger stimulator of the respiration centre than lobeline (1). The presence of cytisine has been stated in a considerable number of plant species. In the literature 51 species of plants have been reported to contain it (2). These are 18 species of *Cytisus*; 10 species of *Baptisia*; 9 species of *Genista*, 4 species of *Ulex*; 4 species of *Sophora*, 3 species of *Crotalaria*; *Gleditschia triacantha*; *Anagyris foetida* and *Euchresta Horsfieldii*.

It it well to point out that in most cases the presence of cytisine was detected by colour reaction with $FeCl_3$ and H_2O_2 [Von de Moer's reaction(3)].

Tse seeds investigated by the present author were those of *Thermopsis lanceolata* (fam. *Leguminosae*). a plant of a wide range of distribution in this country, covering large tracts of land in the steppes of Transbaikal and the Kirghiz republic.

They had been collected in the fall of 1942 on the shores of lake Issyk-kul by an expedition sent to Central Asia by the All-Union Institute of Medicinal Plants under the leadership of P. S. Massagetov.

The grass of *Thermopsis* was subjected to numerous tests in the Alkaloid Department of the All-Union Chemico-Pharmaceutical Research Institute (4-5), and the following alkaloids were isolated: anagirine—$C_{15}H_{20}N_2O$, pahicarpine—$C_{15}H_{26}N_2$, thermopsine—$C_{15}H_{20}N_2O$, homothermopsine—$C_{17}H_{24}N_2O$, methylcytisine—$C_{12}H_{16}N_2O$ and cytisine—$C_{11}H_{14}N_2O$.

What has been a characteristic factor of the previous investigations is that the grass tested did not always contain cytisine (4) or methylcytis ne (5). This is probably because the investigators had to deal with plants collected at different phases of development.

In my tests the seeds of *Thermopsis* have been found to contain 2.13 per cent of total alkaloids and 0.611 per cent of cytisine.

Experimental. 330 g of ground seeds of *Thermopsis* were wetted with 12 per cent NH_4OH and repeatedly extracted with dichloroethane. The dichloroethane extract of alkaloids was then extracted with 5 per cent H_2SO_4. The acid alkaloid solution was passed through a filter and alkalized with 25 per cent NH_4OH. Afterwards followed a reiterated extraction of alkaloids with chloroform.

The alkaloid solution in chloroform was dried by means of calcined Na_2SO_4, and the solvent distilled off on a water bath. Two days later the alkaloid in the form of crystals precipitated from the concentrated solution. Entirely freed from the solvent it was found to weigh 7.03 g. which made up 2.13 per cent of the weight of the tested seeds.

I. Separation of Alkaloids Insoluble in Acetone. The 7.03 g of total alkaloid were treated on heating with 100 ml of acetone. As a result of this treatment I obtained 0.75 g of crystalline alkaloid, insoluble in acetone, which is 0.227 per cent of the initial weight of the seeds. This alkaloid insoluble in acetone is well soluble in water and has a melting temperature of 269—270° C.

From the fact that its melting temperature is so high it may be taken for granted that this is a new alkaloid, not investigated as yet.

II. Separation of Alkaloids Soluble in Ether. The bulk of alkaloids left behind in the acetone solution was entirely freed from the solvent by heating on a water bath. Thereby a thick paste arose which was 6 times treated with ether, each time with a portion of 50 ml. After distilling off the ether, 0.28 g of alkaloids were left behind.

III. Separation of Cytisine in the Form of a Base. The residue of alkaloids left after treatment with ether was dissolved in hot acetone; on cooling needle-like crystals were carried down from the solution; they were found to weigh 0.98 g, or 0.297 per cent of the initial weight of the seeds.

After washing with a small quantity of cold acetone and drying in the desiccator over sulphuric acid, the base showed a melting temperature of 153—154° C (m. p. of cytisine. 153—154°C).

No depression of the melting point resulted from mixing it with cytisine from *Cytisus laburnum* (m. p. 153—154°C).

0.0430 g of the substance fixed 2.22 ml of 0.1 *n* HCl (with methylrot)

0.0512 g » » » » » 2.65 ml of 0.1 *n* HCl.

Molecular weight: found, 194.4; 193; calculated for cytisine $C_{11}H_{14}N_2O$, 190.

Preparation of Cytisine Nitrate. To a concentrated alcohol solution of the separated base HNO_3 was added by small drops until a slight acid reaction to lithmus.

Some time after needle-like crystals of the nitrate fell to the bottom. M. p. 187—188° C (m. p. of cytisine nitrate 187—188° C).

A mixed sample of our nitrate with cytisine nitrate showed no depression of the melting point, but melted likewise at 187° C.

The melting temperature of the base, its molecular weight and the melting temperature of its nitrate permit to identify it with cytisine.

IV. Separation of Cytisine in the Form of Nitrate. The alkaloids that had remained after the separation of cytisine were freed from acetone and dissolved in a small amount of alcohol. To this alcohol solution of alkaloids a concentrated HNO_3 was added by drops. In that way 0.75 g of crystalline nitrate could be separated. The nitrate was recrystallized from alcohol and proved to melt at 187—188° C. Its mixture with cytisine nitrate showed the same melting temperature.

So out of the total of alkaloids I had been able to separate $(0.75 \times 0.721) = 0.54$ g of cytisine in addition in the form of nitrate; which is 0.16 per cent more in terms of the initial seed weight.

V. Separation of Cytisine in the Form of *p*-Toluolsulphanylcytisine. After cytisine nitrate had been separated, the remaining alcohol solution of alkaloids was evaporated in a porcelain cup upon boiling water bath. As the alcohol passed off in vapour, the residue that remained behind was added with 20 ml of water three times in succession. The resulting waters olution of alkaloid nitrates was passed through a filter, and the filtrate was alkalized with 50 per cent KOH until the concentration of KOH attained 6—8 per cent. To the alkaline solution of alkaloids 1 g of *p*-toluolsulphochloride was added, after which the contents was thoroughly shaken and allowed to stand overnight.

The precipitate that formed in the alkaline solution was filtered off and washed first with water and then with sulphuric ether.

After recrystallization from an acetone-ether mixture a *p*-toluolsulphanyl derivative with m. p. 205° C was obtained.

When the *p*-toluolsulphanylcytisine obtained from pure cytisine was mixed with the *p*-toluolsulphanyl derivative separated from the total alkaloid, no depression of the melting point resulted, and this may be regarded as sufficient evidence for identifying the separated compound with *p*-toluolsulphanylcytisine.

By this method 0.80 g of *p*-toluolsulphanylcytisine or 0.44 g of cytisine (0.80×0.552) have been separated in addition, which is another 0.133 per cent of the initial seed weight.

VI. Investigation of Ether Soluble Alkaloids for the Presence of Cytisine. The preliminary treatment of total alkaloid with ether was made with the purpose to separate the alkaloids thermopsine and homothermopsine which are easily soluble in ether. The ether fraction obtained as described above was treated three times in succession with 10 ml of a 12 per cent water solution of KOH. The oil-like residue that refused to dissolve was separated by filtration.

To the transparent slightly yellowish solution of alkaloids thereby obtained 1 g of *p*-toluolsulphochloride was added. With all of the manipulations described in section V only 0.1044 g of toluolsulphanylcytisine could be separated. If recalculated for cytisine this makes 0.06 g or 0.018 per cent of the initial weight of the tested seeds.

Conclusions

1) The seeds of *Thermopsis* are a new source of raw material for the production of cytisine.

2) They have been found to contain 2.13 per cent of total alkaloid, of which 0.611 per cent is cytisine.

All-Union Institute
for Medicinal Plants.

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MINERALOGY

BENTONITE FROM SHOR-SU AND ITS USE IN CAST FOUNDRIES

By S. M. YUSSUPOVA and A. K. USSOVA

(Communicated by V. G. Chlopin, Member of the Academy, 28. VIII. 1943)

The Shor-su deposit is located in the Kaganovich region of the Ferghana district, Usbek SSR, 35 km from the town of Kokand. At Shor-su sulphur and oil have long since been produced.

From the geological standpoint the Shor-su area is composed of Upper Cretaceous, Tertiary and Quaternary deposits. The oldest in the Shor-su deposit is a series of clays and gypsum of Upper Cretaceous age. The most extensively developed within the territory of the region investigated by the authors are Tertiary deposits.

The presence of kil-like clays at Shor-su has been indicated by Uklonsky (1). Later on their existence was confirmed by a number of geologists. No detailed investigations, however, have been as yet carried out on the clays of the Shor-su deposit.

The present paper deals with one of the stages of a large complex study carried on by the authors on the bentonites of this deposit. We give here the results of tests of the Shor-su bentonite clays; these clays may be used as a binder in preparing moulding compounds used in foundry casting.

The main properties of moulding mixes—plasticity, tensile strength, permeability to gases, etc.—depend largely on the quality of the chief binding element of the mix, the moulding clay.

A good moulding clay should be endowed with such a high binding capacity that its addition to the moulding compound in a minimum amount (1 to 4 per cent) be sufficient for imparting to the mix the necessary strength and plasticity.

From the moulding practice it is known that the less the clay content of the moulding mix, the greater the durability and impermeability to gases and the less it is subject to scorching. A clay which answers to the requirements of foundry casting is bentonite.

The binding properties of bentonite depend chiefly on its colloidal state, on its capacity to form a stable suspension with water. A clay with a high colloidal capacity binds a moulding mix the most strongly. The practice of American foundrymen who make extensive use of bentonite has for more than two decades demonstrated the excellent quality of bentonite moulding mixes.

In this Union bentonite has become widely used in foundry casting after the discovery of the Djebel bentonite deposit.

The present authors have carried out a series of investigations and tests in a plant-scale in order to check the applicability of the Shor-su bentonite in foundry casting as a binder. With this object in view, the authors used specimens of raw material from the 2nd anticline which is the most convenient for working.

In the first place it was necessary to find out whether the clay from this particular deposit really belonged from its mineralogical composition to the group of bentonites.

X-ray examination gave a typical X-ray photograph of montmorillonite: d (hkl)=7.42; 4.28; 3.00; 2.54; 1.46; 1.27; 1.24. The thermographs and dehydration curves show a thermal effect characteristic of montmorillonite (Fig. 1, 2 and 3).

As it will be seen, the thermographs show two thermal effects: the first between 105 and 200°, and the second, between 600 and 750°. Upon dehydration the

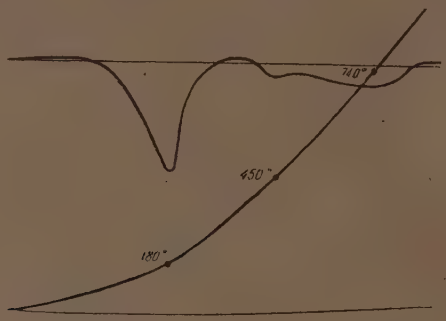

Fig. 1. Shor-su. Specimen No. 13. Thermograph

bulk of water is evolved at a comparatively low temperature (105—200°), then follows a smooth and insignificant evolution up to a temperature of 500°. Between 500 and 700° there is again a sharp increase in the evolution of water. These data confirm the montmorillonitic composition of the specimens studied.

The chief properties of bentonite clays which serve as a criterion for establishing their fitness for casting purposes, is the value of the coagulating capacity (the bentonite number) and the swelling capacity.

The bentonite number was determined by the method commonly applied.

A 5 g sample of air-dry clay is mixed with 0.5 per cent magnesium oxide, covered with water, thouroughly shaken for 15 minutes, and left in a graduated

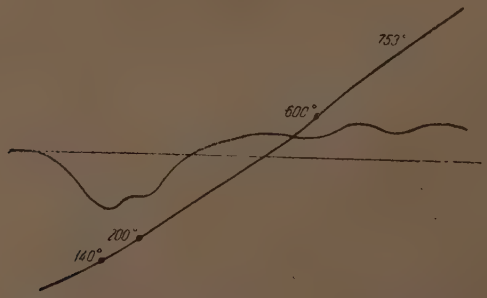

Fig. 2. Shor-su. Specimen No. 14. Thermograph

cylinder for 24 hours. Upon settling, the ratio of the height of the suspension column to the total height of the system (precipitate + sediment) is calculated. This ratio expressed in per cent gives the value of the bentonite number. Good bentonites give a sufficiently compact mass which does not pour out when the vessel is held upside down. Clays having a bentonite number not lower than 40—50 are considered to be suitable for foundry purposes. The samples studied by the authors had the following bentonite numbers:

Sample	13	14	15	22	31
Bentonite number	75	98	57	48	27

The best bentonites from Oglanly, according to the authors' data, have a bentonite number equal to 83. Thus, the Shor-su bentonite from its coagulating capacity is in no way inferior to the Djebel bentonite.

The swelling capacity was determined in the following manner. In graduated test-tubes an air-dried weighed portion was taken, covered with water and sha-

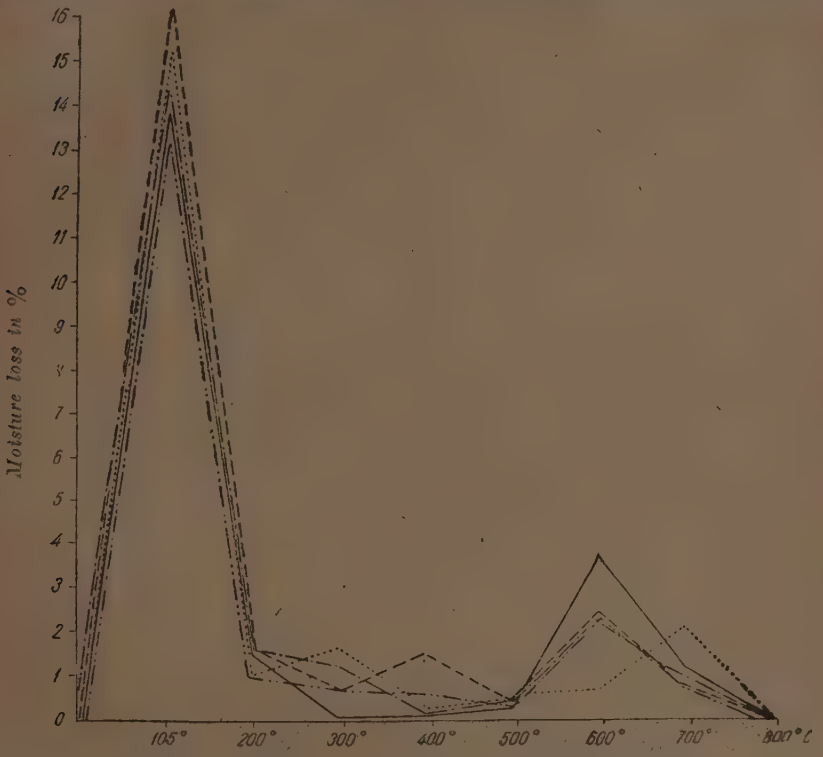

Fig. 3. Dehydration curves of Shor-su clays. Second anticline
— sample 13; — — — sample 14; · · · · · sample 15;
— · — · — sample 22; — · · — sample 31

ken for 10—15 minutes. Every 24 hours the increase in volume was recorded, the initial volume being taken as 100 per cent.

The specimens studied by the authors had an exceptionally high swelling capacity: the minimum swelling value was found to be 470 per cent, and the maximum, 1111 per cent, viz.:

Specimen	13	14	15	22	31
Swelling capacity in %	818	1000	1111	550	470

Such a large swelling value is evidence of the montmorillonitic composition of the clays and, consequently, of these rocks belonging to bentonites.

One of the essential properties characterizing the quality of the bentonites is their plasticity. A determination made after Atterberg gave the following results:

Specimen	13	14	15	22	31
Plasticity	43.0	54.8	52.6	22.0	32.8

As it will be seen, the plasticity of the samples studied by the author attains 54.8. Such a high plasticity is peculiar to clays with a montmorillonitic compo-

sition (2), which accounts for the high physical-mechanical characteristics of the clays studied.

The physical-mechanical tests were made at the laboratory of the Voroshilov «Selmash» works. The moulding mixes are prepared at this works on Darbaz sand which was also used by the authors in their tests.

The sand was added various admixtures of Shor-su bentonite. The mixture was thoroughly agitated on rollers for 10—15 minutes, after which the permeability to gases, compression strength, and tensile strength were determined. As it will be seen from the table, the Shor-su bentonite has high physical-mechanical characteristics, being by no means inferior to the Oglanly bentonite.

% of bentonite added	Permeab. to gases, cm/min.	Compression strength, kg/cm²	Tensile strength, kg/cm²	Moisture content, %
Sand	135	—	0.05	1.20
1.0	135	0.18	0.09	2.50
2.0	140	0.23	0.15	3.20
3.0	140	0.30	0.19	3.80
4.0	130	0.45	0.23	4.75
5.0	90	0.58	0.37	5.40
6.0	70	0.62	0.48	5.70
7.0	55	0.65	0.53	6.00
8.0	33	0.72	0.65	6.50
9.0	23	0.75	0.90	7.20
10.0	10	0.75	1.40	7.70
3.5 Oglanly bentonite	152	0.52	—	2.00(3)

An increase in the percentage of bentonite added results in the improvement of the physical-mechanical characteristics; except for the permeability to gases. With an increase in the percentage of bentonite, the permeability to gases decreases. Within 1—3 per cent here takes place an increase of the permeability to gases, followed by a slow decrease. In casting an addition of more than 4—4.5 per cent of bentonite is not practised. The decrease of permeability to gases cannot therefore be taken to impair the quality of a given bentonite. Such a reduction of permeability to gases is no occasional one, but is the result of a filling of the pores of voids in the sand mix.

The data obtained point to the fact that the clays studied belong to the bentonite group, and can therefore be used for foundry purposes. The proximity of the deposits to the industrial enterprises of Uzbekistan makes unnecessary transportation, thus increasing the possibility of a wide application of the bentonites to other branches of the national economy.

An approximate estimation of the reserves and the mineralogical, physical and mechanical characteristics of the deposit studied give us ground to believe that the metallurgical industry of Uzbekistan will be supplied with a necessary quantity of bentonite clays from the Shor-su deposit.

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of the Academy of Sciences of the USSR.

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MICROBIOLOGY

ACCUMULATION OF VITAMIN B_1 BY YEAST CELLS

By E. N. ODINZOVA

(Communicated by L. A. Orbeli, Member of the Academy, 10.VI.1943)

In a previous work (1) vitamin B_1 has been described by us as a most essential factor in the expression and development of the fermentative function of yeast organisms. Thus, observing the changes it undergoes in the cell; we became aware of the disproportion between the content of this vitamin as observed in the case of fermentation or in cultures developing in aerobic conditions. Whereas the yeasts *Saccharomyces* (*Sacch. cerevisiae*, *Sacch. Carlsbergensis*) were found to contain as a result of fermentation up to 200 micrograms of vitamin B_1 per 1 g of dry substance, when propagating under conditions of good aeration the amount of vitamin B_1 in them was as low as 5 micrograms per 1 g of dry substance of the yeasts. From a comparison of diverse forms of yeasts and yeast-like organisms differing as to the type of fermentation produced by them; the same regularity could clearly be borne out, viz. the content of vitamin B_1 under similar propagation conditions is well correlated with the fermentative activity of a given organism.

In our short experiments on fermentation (on sugar phosphate solution; with a great number of cells*) we succeeded in bearing out with a particular clearness the liability of yeasts to accumulate vitamin B_1. Within the short period of fermentation (60 min.) vitamin B_1**, when introduced into the medium; was found to accumulate in the cells in very large amounts: up to 1000 micrograms per 1 g of dry yeast substance. This seems to be the limit of saturation of a microbic cell with vitamin B_1.

How excessive this figure is, may be seen from a comparison with various animal and vegetable tissues. Animal tissues known as being high in vitamin B_1 contain on the average up to 10 micrograms; vegetable tissues up to 50 micrograms of vitamin B_1 per 1 g of dry substance.

We have also observed the accumulation of vitamin B_1 by yeasts from natural substratum under conditions of short-lived fermentation.

At the same time with the present author; similar results have been obtained by Fink and Just(4) and Sperber and Renvall(5).

Further experiments with yeast-like organisms, *Torula utilis* and *Endomyces Magnusii*, enabled us to conclude that only a small part of the vitamin B_1 accumulated directly participates in fermentation. The greatest part seems to be deposited in the cell at the period of fermentation, like the reserve nutrient products, fat; glycogen, etc.

The experiments were carried out in the following way. *Torula utilis* or *Endomyces Magnusii* were being cultivated for 48 hours on a solid agar mineral me-

* Under these conditions multiplication is almost lacking.
** Crystalline, of the firm Hoffmann La Roche.

dium (Reader) with sugar. The medium was added various amounts of vitamin B_1. *Torula utilis* was found to synthesize vitamin B_1 in insufficient quantities, while in *Endomyces Magnusii* the synthesis of vitamin B_1 was almost totally lacking ([1–3]). If at the period of propagation the medium contained insufficient amounts of vitamin B_1, both forms would show a reduced fermentative activity. When such a culture was taken for short-lived fermentation, an addition of vitamin B_1 invariably produced a stimulating effect.

Cultivation of *Torula utilis* and *Endomyces Magnusii* under conditions of deficiency or excess of vitamin B_1 showed that reduced fermentative activity was observed in those cases only when the concentration of the vitamin B_1 in the medium was below 0.5 micrograms per 1 cm^3 of the medium. Beginning with a concentration of 0.5 micrograms of the vitamin per 1 cm^3 of the medium and above, the fermentative activity reached its limit and remained constant, even with rather high concentrations of the vitamin B_1 in the medium and, consequently, in the cells, too. It should be mentioned that no reaction to the vitamin B_1 added during the short-lived fermentation test could be observed.

The fermentative activity of cultivated yeasts, various species of *Saccharomyces*, also remained stable when vitamin B_1 was accumulated in quantities ranging from 20 micrograms up to 1000 micrograms per 1 g of dry yeast substance.

The experiments described make it obvious that quite insignificant quantities of vitamin B_1 are sufficient for a yeast cell to produce fermentation. The excess of the accumulated vitamin B_1 evidently remains inutilized.

Yeast-like organisms may thus be considered as powerful biological agent capable of accumulating the vitamin B_1 scattered in nature and, probably, many other analogous substances, and of concentrating them in large quantities in their cells.

The accumulation of crystalline vitamin B_1 introduced in the medium was also observed by us under aerobic conditions of propagation. However, its concentration in this case was quite insignificant; if compared to that observed in the case of fermentation. We could determine no more than one tenth of the total vitamin B_1 introduced into the medium. And very little was found to remain in the medium; too.

The ability of cultivated yeasts to accumulate large amounts of vitamin B_1 may be turned to profit for obtaining highly active natural polyvitaminous preparations.

It appears of great interest to find out what is the principal rôle played by vitamin B_1 at the period of development and propagation of yeast cells. We may assume that the vitamin is to a certain extent spent here as a plastic material. But this cannot account for the large quantities of vitamin that are observed to disappear from the medium at the period of active propagation of the culture, when the latter is well provided with oxygen from the air.

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PLANT PHYSIOLOGY

STERILE CULTURES AS A METHOD FOR STUDYING METABOLISM IN HIGHER PLANTS

By W. O. TAUSSON, A. A. PROKOFIEV and W. E. PONTOVICH

(Communicated by A. N. Bach, Member of the Academy, 5.VI.1943)

The study of many problems of metabolism in higher plants by the usual method of water, sand or soil cultures proves a rather difficult matter, and there are cases when it is altogether impracticable because of the interference of various micro-organisms. This is especially true of experiments on the heterotrophic nutrition of green plants through the root system, and of such studies as are dealing with some problems of metabolism by striking into the balance matter and energy. Studies of the latter type, which gave highly interesting and valuable results with lower plants (moulds) [1–3] are likely to be no less successful with higher plants too, provided that the interference of microbes be entirely eliminated.

In order to answer a whole array of questions in the domain of physiology and biochemistry of nutrition and metabolism in higher plants, most various organic substances are to be introduced into the substratum; most of these substances are liable to change rapidly or to be destroyed under the influence of micro-organisms.

It stands to reason therefore that in order to obtain reliable and irrefutable results, experiments or procedures of any kind involving the application of nutrient media containing organic substances, should necessarily be carried on under sterile conditions.

This implies a fully reliable, sufficiently simple and convenient method of sterile cultivation of higher plants.

Such methods of sterile cultures of separate parts and organs of higher plants have been worked out by number of students and are now being used on a wide scale.

To these belong works on separated embryos [4–5] and isolated roots [6–8]. However, the cultivation of separate isolated organs is attended by certain difficulties and is far from being always a success because of our insufficient knowledge of the supplementary factors of growth operative here [9–11]. But even in the case of a vigorous and prolonged growth of the roots isolated (say, in tomato plants), their development is never quite normal in all respects [11–13]. Along with a number of other reasons and considerations, this brings to the fore the study of the effect produced by various organic substances upon an integer plant both in light and in the dark, which cannot be done unless sterile conditions are guaranteed.

The method of sterile cultures had been worked out by Shulov [14] and Tanaka [15] and was successfully applied by these writers. However, the method is somewhat too complicated and requires special glass vessel, which prevents it from finding general application in laboratories for mass experiments and cultures.

The method worked out by the present authors is requiring neither special apparatus nor vessel; at the same time it makes it possible to deal with many questions of metabolism in higher plants and to strike the balance of substance and energy.

The first thing to do when starting a plant culture under sterile conditions is to destroy entirely all microflora on the seeds prior to sowing. The sterilization of the seeds is brought about by treating them with various substances (corrosive sublimate, alcohol, bromine, water, etc.). The possible injurious influence of the sterilizer applied on the seeds and sprouts should, however, always be borne in mind.

Numerous tests have shown that quite reliable results are obtained by treating the seeds with a 10—12 per cent H_2O_2 solution during 20 minutes *. The insignificant traces of H_2O_2 that remain on the seeds after washing do them no harm and are soon decomposed.

As a sterilization apparatus may serve a thick-walled test-tube supplied with a well-fitted rubber stopper through which a glass tube with a tap is introduced.

The seeds are put into the test-tube by small portions (40—100 seeds at once) and a H_2O_2 solution of the concentration specified above is poured over. After the test-tube has been stopped, its contents are shaken during a definite period of time; the excess of oxygen is sometimes to be let out through the tap. As soon as sterilization is over, the liquid is poured out over the brim of the vessel, with due precautions taken, and the seeds are washed under sterile conditions in several portions of sterile water. Together with the last portion of water the seeds are poured into a sterile Petri dish.

In case planting is to be done with germinated seeds; the bottom of the Petri-dish is to be covered prior to sterilization with one or two layers of filtering paper. This procedure can be omitted if planting is to be done directly upon sterilizing.

For sterile plant cultures common laboratory vessel may be used: Ehrlenmeyer test-tubes and bulbs of various dimensions. If the cultures are grown on solid substrata (agar nutrient media; pumice moistened with a corresponding nutrient solution, etc.), then the vessels listed above need no additional appliances.

In the case of cultures grown in liquid media, the installation of special capsules is necessary.

In test-tubes the capsules are adjusted in the following way: in a test-tube 30 cm^3 in volume a rubber plate of a corresponding diameter is inserted; the plate is perforated in the centre; and into the opening a glass capsule with a tapering lower end is inserted.

The seedling is placed into the capsule whose size may vary as to that of the seedling or seed. The rubber plate is so inserted into the test-tube that only the rootlet be immersed in the nutrient solution**.

If the cultures are grown in large vessels (for example, in Ehrlenmeyer bulbs of a 1—1.5 l capacity), the capsules are adjusted in the following way: when manufacturing a cotton stopper for the bulb, cotton is wound on a glass rod having a swelling at its lower end. This swelling serves as a support for the perforated rubber circle into whose opening the capsule is introduced. By lifting or sinking the glass rod one is able to regulate the immersion of the roots into the medium.

When several capsules are inserted into the rubber circle, several plants can simultaneously be grown under perfectly similar conditions.

Vessels prepared in this way are to be sterilized together with the nutrient media they contain.

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* If sterilization is applied to separated embryos, the treatment is made to last several times less (no more than 4—5 minutes).

** The seedlings should not be immersed too deep into the medium, the growth of the plant being in this case either strongly delayed or stopped altogether.

The composition of the nutrient media way be varied according to the kind of the experiment. For investigating the fitness of sources of carbon and for some other purposes the following modification of Knop's medium should be recommended: $Ca(NO_3)_2$, 0.1 g; KNO_3, 0.025 g; $MgSO_4$, 0.025 g; $Fe_2(SO_4)_3$, 0.001 g; H_2O, 100 cm³; KH_2PO_4 and K_2HPO_4, 0.025 g.

When studying the influence of various organic substances on the growth and development of plants, the former are added to the medium at a due concentration, viz., sugars up to 2 per cent, organic acids from 0.002 to 0.05 per cent (according to their chemical structure). Organic nitrogen (asparagin, diverse aminoacids, etc.) are to be introduced at concentrations corresponding to the purposes of a given investigation. Whatever the experiment, special attention should be paid to the choice of a pH that would suit the plant experimented upon. Thus, for example, some of the plants studied by us (root rubber-plants and especially tau-saghyz) gave no satisfactory growth outside of a narrow range of pH from 7.0 to 7.8.

Particularly susceptible to the acidity of the medium are plants cultivated in the dark on carbohydrates.

The degree of acidity required may easily be attained by duly combining acid (KH_2PO_4) and alkaline (K_2HPO_4) phosphates (buffer mixture according to Sörensen).

When using media with phosphate buffers, these are to be sterilized separately and added to the medium upon sterilization*.

The provision of plants with supplementary growth factors which are of particular importance in the case of cultures grown in the dark is attained by adding to the medium extracts[17] of yeasts or lower fungi. Reinforced diet containing metabolic products of some living fungi also proves very efficient.

A higher plant and a fungus can be cultivated jointly in an appliance of the following type: into a wide-mouthed Ehrlenmeyer bulb provided with a cotton stopper a glass tube is inserted with a small constriction at its lower end. The upper tip of the tube is stopped with the cotton stopper, the lower is covered with cellophane fixed with a solid thread. Both the bulb and the tube are filled with a medium of a composition required. Upon sterilization sowing is practised, the seeds or the isolated root-tips being planted into the bulb, and the spores of the fungi, into the tube.

It should be taken into account that after two or three weeks of cultivation the spores of the fungi may grow through the cellophane and directly produce their harmful effect upon the plant. The tube with the fungus is therefore to be renewed at regular intervals (every 8—12 days), according to the nature of the fungus, under conditions of perfect sterility. The simplest procedure is to take a tube stopped as it is from another bulb with a similar mouth diameter as that of the experimental bulb containing the culture.

When sterilizing the media it should be borne in mind that some organic substances are highly susceptible to high temperatures and are decomposed if treated in an autoclave.

In this case Koch's method of sterilization with current steam is to be applied.

The technique of sowing is a rather simple one: from the Petri dish the seeds are transferred with the help of sterile pincers into the capsule of the experimental vessel; all sterility conditions being strictly observed.

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* The phosphates are added either by a direct introduction of about 5.0 per cent of the solution with the aid of a sterile pipette, or by pouring a less concentrated phosphate solution together with the main medium. In the former case a graduated pipette may conveniently be sterilized together with the test-tube containing the phosphates, having adjusted it to the cotton stopper of the test-tube. Some other methods of adding the phosphates are described elsewhere by one of the authors [16].

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EVOLUTIONARY MORPHOLOGY

WHAT ARE THE PARTS OF THE REGENERATION TERRITORY OF A FORE EXTREMITY NECESSARY FOR ITS RESTITUTION

By M. I. EFIMOV

(Communicated by L. A. Orbeli, Member of the Academy, 13.VI.1943)

If a fore limb is amputated together with its shoulder girdle, the wound area gives development to skin alone, while the deficient parts do not regenerate. The remaining tissues, i. e. those of the wound surface, may give rise to a regeneration process of an extremity and of its shoulder girdle, provided their condition and relations be changed (1).

Similar results may be obtained by way of grafting upon the wound area parts of both a given territory [skeleton, muscles, corium (1)], and parts of other territories [skeleton; corium(2)].

The data obtained lead us to the question as to what parts of the regeneration territory may be substituted by parts of different territories, and what of them are indispensable for the restitution of the limb and its shoulder girdle.

The regeneration territory of a fore extremity is composed of the skin, the skeleton and the muscles. The participation of the skin in the regeneration process is indispensable (3); the skin of the fore limb may, however, sometimes be replaced by skin from other territories, without changing the results of the development (4).

The regeneration process of a fore limb and the shoulder girdle may go on under conditions of a total absence of the skeleton of the extremity and its shoulder girdle (5, 6), but a transplanted skeleton of the shoulder girdle (1), or even the skeleton of other territories (2), is liable to restitute the regeneration capacity of a given territory.

The question as to whether the restitution of a fore limb and of the shoulder girdle may take place without the participation in this process of the musculature of the fore limb territory, still lies open and requires further investigation.

In order to solve the questions referred to above, the following special experiments were carried out. Their aim was to produce a regeneration process of an extremity by means of transplanting a fore limb on to a different territory, and in this way to find out, what are the parts of a given territory that are indispensable for its restitution.

As material for these experiments use was made of white and black axolotls, from 6 to 12 months old. The axolotls were kept in aquaria, several in each, at indoor temperature, and fed on meat.

Run No. 1. This run involved grafting of the skeleton of the shoulder girdle under the skin of another regeneration territory. Grafting was practised on to

the caudal fin, i. e. a region deprived of muscles. In controls the skeleton of the shoulder girdle was removed, being carefully freed of the corresponding muscles. The experimental axolotls were grafted through a cutaneous incision the skeleton of the shoulder girdle. A total of 50 grafts were performed.

At various time intervals (ranging from half a month to one month and a half) the skin in front of the joint socket was removed, a wound area exceeding in size that of the humerus diameter thus being created. In a considerable number of axolotls the wound surface was formed spontaneously, as a result of a gradual decay of the skin covering the joint socket. In the subsequent a small tubercle was formed in front of the joint socket, the graft showing a strong hyperaemia.

The tubercle newly formed was of the type of a blastema; it showed but slight signs of growth, according as the graft was getting smaller in size. In some cases the tubercle persisted during several months, and disappeared along with an almost total disintegration of the graft.

At the place of the epithelial membrane that covered the tubercle; skin was found to develop.

In other cases, in front of the joint socket another tubercle was formed, of the type of a blastema, but it failed to develop further. After some time the tubercle became overgrown with skin, the graft being preserved. In either case there was no regeneration process. Observations were carried on during 3—6 months.

Run No. 2. This run differed from the preceding one only in that along with the transplantation of the shoulder girdle skeleton, skin from the territory of the extremity was also transplanted. The operation was made at two times. At first the skin at the spot of transplantation was removed and instead of it skin from the territory of the limb was transplanted. After some 15—20 days the skeleton of the shoulder girdle was grafted under the skin in such a way that its joint socket be in front of the central part of the graft skin. After the two grafts healed in, a wound surface was created in front of the joint socket of the skeleton of the shoulder girdle, by partly removing the skin, whose area exceeded the diameter of the humerus.

On the wound surface the same alterations could be observed, their issue being the same as in the previous run.

Run No. 3. This run involved transplantation of the skeleton of the shoulder girdle together with areas of caudal muscles under the skin of the caudal fin.

A total of 9 axolotls were operated upon. Upon healing-in the graft, in front of the joint socket of the skeleton of the shoulder girdle a wound surface was created by removing the skin. In some cases the wound area gave rise to the development of skin, in others, of a small outgrowth consisting of fin-like tissue. The regeneration process was lacking in all cases.

Run No. 4. This time I grafted skeleton and muscles of the shoulder girdle under the skin of the caudal fin. After 20—25 days the skin in front of the joint socket of the skeleton was removed, a wound surface thus being created. A total of 9 axolotls were operated upon. In one case there was development of an atypical extremity. In six cases regenerates were developed showing at the same time parts of extremities and a fin-like mass (chimeras). In two cases the wound area was found to overgrow with skin.

The data of this experimental run testify to the fact that the skeleton and the muscles of the shoulder girdle, when grafted on to a different territory, are able to produce in their new place a regeneration process of the extremity. The atypic features of the regenerating limb, or the appearance of chimeral regenerates, are the consequences of the participation in the process of local tissues.

A comparison of the data of this run with those of the preceding three runs makes us arrive at the following conclusions.

Conclusions

1. The skeleton of the shoulder girdle combined with skin from other territories, irrespective of the presence or absence on it of muscles of other territories, is unable to secure a regeneration process of the extremity.

2. The skeleton of the shoulder girdle, together with the skin of the territory of the fore limb, gives no regeneration process of the extremity, irrespective of the presence or absence of muscles of foreign origin.

3. The skeleton of the shoulder girdle combined with its muscles, in the case skin of foreign origin is present, leads to the formation of an atypical extremity or of a chimeral organ.

4. The participiation of muscles of the regeneration territory of the fore limb in the process of restitution of the extremity is obviously indispensable.

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ICHTHYOLOGY

PREFERRED TEMPERATURES OF FISH LARVAE

By N. V. EVROPEYZEVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 18.V. 1943)

Preferred temperatures, or the thermotactic optimum were studied chiefly in insects ([1],[5]) and mammals ([6]). A few papers only are devoted to the investigation of the thermotactic optimum in marine invertebrates([2],[4],[7]), and there is but one concerned with fishes ([3]). There are no literary data whatever on the question of the preferred temperatures of fish larvae. The importance of such a study is obvious, especially if we consider the fish breeding procedures widely applied in this country, and the poor state of knowledge of the ecology of fish larvae.

In the experiments described below burbot larvae obtained from artificially fertilized eggs were used. The eggs were fertilized at the river Shouya (lake Onezhskoye), on the 12th of February, 1939, and transferred on the 23rd February to the Peterhof Biological Institute where the experiments were performed. For these experiments a somewhat modified Guerter apparatus was used which had already been applied at this Laboratory for earlier investigations([2],[4]).

The experimental procedure was as follows. In a vessel with a 1—1.5 cm layer of water, prior to establishing the gradient, a definite number of larvae were introduced; these were more or less regularly distributed throughout the divisions of the apparatus. Readings were taken one or two hours after the gradient had been established and the larvae distributed, every 10 minutes in the first five experiments and every 5 minutes in the rest. The apparatus was graduated into 100 divisions of 0.01, from 100 to 0, and 10 divisions of 0.1, from 10 to 0, the thermometers being set 0.2 apart. At each reading the number of larvae was determined for every large division (0.1). When treating the results obtained, temperatures for intervals of 1 degree were determined by graphical interpolation. For plotting the frequency distribution with temperature intervals of 1 degree, the number of larvae was counted for each centisimal division of the apparatus; it was assumed thereby that the larvae were regularly distributed within every tenth division. Then the total of the readings corresponding to one tenth division of the apparatus were divided by ten, and the figures thus obtained distributed among the temperature intervals of the apparatus at every other degree. This method involves some inaccuracy which plays no essential part in this case, on account of the larvae being highly active.

The table represents the thermal conditions of the embryonic and post-embryonic development of the burbot, giving a short morphological characteristic of the experimental larvae and the essential elements of frequency distribution; a general idea of the mean preferred temperature may thus be gained. It should be pointed out that the larvae hatched at various terms and showed various degrees of development. The two first experiments were run with larvae hatched

under natural conditions. As may be seen from the table and Fig. 1, the larvae of this stage prefer temperatures ranging from 3 to 6°C, about 50 per cent of their total number being concentrated within this interval. The curves look somewhat incomplete because of the rather short range of temperatures applied in the first experiments. In experiments 3, 4, 5 we used larvae that hatched on April 25, 1939, and were developing in aquaria with running water.

Upon resorption of the yolk-sac larvae were fed on a culture of *Paramoecia*. Experiment 3 gives a picture similar to that of experiments 1 and 2 (Fig. 2), while in experiment 4 a marked shift of the preferred temperatures is observed: more than 50 per cent of the larvae are found to be concentrated between 6 and 12° C, and the mean preferred temperature rises up to 9.5° C. Experiment 5 is characterized by a still greater rise of the preferred temperatures: about 60 per

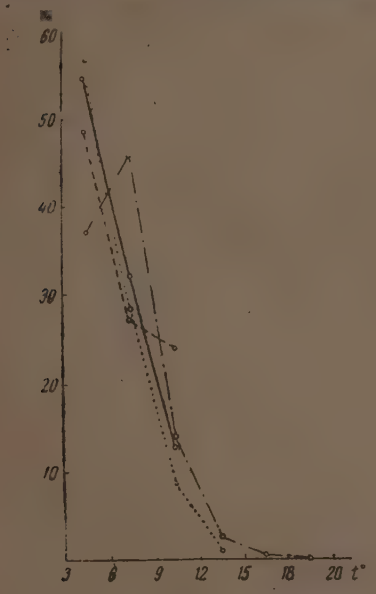

Fig. 1.—exper. 1; — — — exper. 2; ..... exper. 8; —·—·— exper. 9

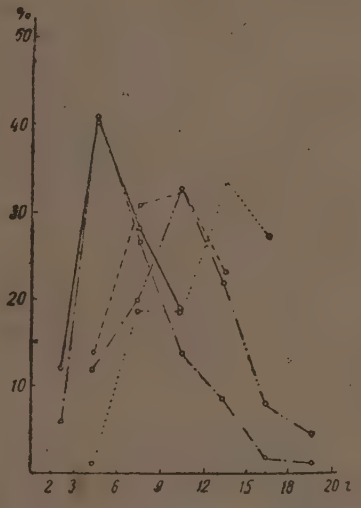

Fig. 2.—exper. 3; — — — exper. 4; ..... exper. 5; —·—·— exper. 6; —··—··— exper. 7

cent of the larvae were found to concentrate between 12 and 16° C. Thus, three consecutive experiments with the same lots of larvae showed a gradual increase of the preferred temperatures within four days. Experiment 6 was carried out with the larvae of the preceding test, kept for 5 days at a lower temperature (4.4° C). The table and Fig. 2 show a distinct drop of the preferred temperatures: about 74 per cent of the larvae concentrate within the temperature interval of 2 and 9° C. The results of experiment 7 with larvae hatched on April 19, 1939, are very similar to those of experiment 5 (table and Fig. 2). The last two runs, 8 and 9, involve larvae hatched on April 13, 1939, kept after hatching at low temperatures (see table). Though 33-day-old, their stage of development was about the same as in experiment 3, which should be put down to a reduced rate of development. In both experiments a clear-cut reaction of the larvae to temperature may be observed with a well-pronounced preference to lower temperatures, like in the first three experiments.

When analysing the results of the experiments described, the question presents itself as to what degree the data secured are illustrative for the actual reaction of burbot larvae to temperature, and for the choice of preferred temperature under

Nos. and date of experiment	Age of larvae in days and date of hatching	Temperature of eggs development in °C		Temperature of post-embryonic development		Size of larvae in mm		Morphological characteristics	Range of gradient in °C	n	$M \pm m$	σ
		range of fluctuation	mean	range of fluctuation	mean	range of fluctuation	mean					
No.1 4.V.1939	2 2.V.1939	—0.5÷+8.2	1.6	8.2÷9.6	9	4.05÷4.26	4.15	I stage*. Vestiges of yolk-sack present. Large oil drop. Mouth open. Gill filaments rudimentary. Blood vessel system well developed. Intense pigmentation	3÷12	330	6.56±0.12	2.22
No.2 5.V.1939	3.5 2.V.1939	—0.5÷+8.3	1.6	8.2÷9.6	9.2	4.05÷4.26	4.15	Same	3÷12	450	6.70±0.13	2.
No.3 5.V.1939	11 25.IV.1939	—0.5÷+5.2	1.3	4.2÷9.0	6.8	4.26÷4.32	4.28	II stage. Yolk-sac entirely resorbed. Oil drop reduced in size. Mouth terminal. Gills functionning. Intense pigmentation	2÷12	300	6.21±0.18	3.03
No.4 7.V.1939	13 25.IV.1939	0.5÷+5.2	1.3	4.2÷9.0	6.9	4.28÷4.34	4.29	Same	3÷15	300	9.5±0.18	3.12
No.5 8.V.1939	14 25.IV.1939	—0.5÷+5.2	1.3	4.2÷9.0	7.0	4.28÷4.34	4.29	Same	3÷15	300	12.42±0.22	3.87
No.6 13.V.1939	19 25.IV.1939	—0.5÷+5.2	1.3	up to 8.V.1939 4.2÷9.0	7.0	4.28÷4.35	4.30	Same	3÷20	435	7.21±0.17	3.53
				8—13.V 4.2÷5.0	4.4							
No.7 16.V.1939	27 19.IV.1939	—0.5÷+3.8	1.1	6.2÷11	8.8	4.44÷4.58	4.54	II stage. Insignificant oil drop. Mouth terminal. Blood vessel system well developed. Intense pigmentation	3÷21	560	10.79±0.16	3.85

Continued

Nos. and date of experiment	Age of larvae in days and date of hatching	Temperature of egg development in °C		Temperature of post-embryonic development		Size of larvae in mm		Morphological characteristics	Range of gradient in °C	n	$M \pm m$	σ
		range of fluctuation	mean	range of fluctuation	mean	range of fluctuation	mean					
No.8 16.V.1939	33 13.IV.1939	—0.5÷+3.0	1.2	1.4÷8.4	3.7	4.08÷4.32	4.16	II stage. The larvae have a large oil drop. Otherwise correspond in development to 11-day-old larvae of experiment 3	3÷22	600	5.9±0.08	2.08
No.9 17.V.1939	34 13.IV.1939	—0.5÷+3.0	4.2	13.IV—13.V 1.4÷8.4 14—17.V 8.8÷11.0	3.5 10.1	4.08÷4.32	4.16	Same	3÷22	600	6.54±0.14	3.64

* The first stage is characterized by the presence of the yolk-sac and by a passive mode of feeding.
The second stage comprises the period following the resorption of the yolk-sac and till differentiation of unpair fins sets in, and is characterized by a transition to active feeding.

natural conditions. In nature the embryonic development of burbot larvae proceeds under the ice, i.e. at a temperature of about 0°. Incubation with burbot eggs showed that the most favourable temperature for embryonic development ranges from 0 to 4° C. It seems that in nature burbot larvae hatch soon after the water is cleared of ice, i. e. when the temperature of water is subject to very rapid variations. It is therefore natural to expect in larvae of different stages an adaptation to such changes, as well as differences in their preferred temperatures. Temperatures from 2 to 4.5° C proved fatal to larvae at the second stage of development (approximately in the middle) under artificial conditions. Larvae of this stage developed normally at temperatures of 8—10° C. In nature (lake Onezhskoye) burbot larvae at a stage corresponding to the second one were caught the 21st of May at a temperature of 9.6°C, and the 28th of May, at 13.8° C.

Summing up all these data, the following conclusions can be arrived at: 1) The gradual rise of the preferred temperatures observed in our experiments (experiments 3, 4, 5) reproduces the actual course of adaptation to rising temperatures under artificial conditions. Our experiments might possibly have involved the very period of development (second stage); when a shift towards higher preferred temperatures takes place. 2) Within the first days upon hatching (first stage, beginning of the second one) the reaction to temperature is more distinct (experiments 1, 2).

It should be noted that the statistical frequency mean cannot fully express the reaction of temperature preference, as has already been remarked (1).

In order to give a complete picture of the temperature preferred it is necessary to give also the whole range of temperatures to which a given organism dis-

plays preference at a given period. And it seems scarcely possible to agree with the conclusions arrived at by Doudoroff ([3]) who believes experimentally established preferred temperatures to be incomparable with those observed under natural conditions, the latter being due to the reaction of keeping away from extreme temperatures, developed by fishes. This question, however, requires further investigation.

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PALAEONTOLOGY

AN ATTEMPTED RESTORATION OF *PSAMMOLEPIS PARADOXA*

By Dm. OBRUCHEV

(Communicated by A. A. Borissiak, Member of the Academy, 22.V.1943)

From the Psammosteids only *Drepanaspis gemündenensis* has been repeatedly reconstructed (1), owing to its finding in form of whole though compressed and deformed armours. Other members of this family are met with in the form of fragments, at the best of separate plates of the armour.

Fig. 1. Restoration of *Psammolepis paradoxa*: *a*, ventral view; *б*, dorsal view. $^1/_3$.

The only attempt to figure the structure of some other Psammosteid is a drawing by Gross [(2), Pl. 1, Fig. 1], representing a transverse section of *Psammosteus* (at that time this genus was not yet divided in two: *Psammosteus* s. str. and *Psammolepis*). However, this reconstruction was afterwards abandoned by the author himself. It was a combination of a strongly convex ventral plate of

Pycnosteus palaeformis; in the rôle of the dorsal plate, and a ventral plate of *Psammolepis paradoxa*, in the rôle of the ventral; on either side were protruding branchial plates of the latter species.

Although *Psammolepis paradoxa* (Middle Devonian of the Baltic states), owing to the works by Gross; is known better than any other Psammosteid from our Devonian, its restoration is very difficult, since a great part of the plates and their interrelations are unknown. The restoration is drawn by N. A. Yanshinov after my sketches; a plaster model is made by Y. M. Eglon.

The dorsal plate (Fig. 1,*б*) is outlined after Gross' drawing [(²), Fig. 1A]. The ventral (Fig. 1,*а*) is restored; since in the specimen figured by Gross [(²), Pl. 1; Fig. 3] the front and hind ends are lacking; the front end is restored approximately after the pattern of *Drepanaspis* and *Schizosteus*; at the hind border only a small notch is inserted, contrary to the genera *Drepanaspis* and *Schizosteus* (¹). In these the notch is very deep; reaching sometimes to the ossification centre. In the preserved portion of the ventral of *Psammolepis* there are no traces of such a notch. Probably, this genus was nearer *Psammosteus* in the structure of the ventral.

The branchials are outlined after Gross' drawings [(²), Pl. 1, Fig. 6; (³), Fig. 1B]; their relations to the large median plates are deduced from the outlines

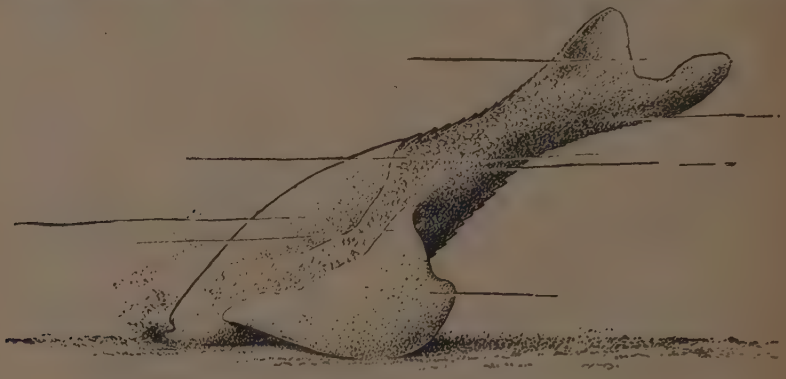

Fig. 2. Restoration of *Psammolepis paradoxa*, side view. 1/3.

of their medial borders which repeat somewhat the contours of these plates. The zones of small plates between the larger ones are drawn by analogy with *Drepanaspis* as well as the form of the tail and the position of the mouth and the eyes. Other larger plates corresponding to the orbitals, cornuals etc. of *Drepanaspis*, are not shown, since their form and position are totally unknown.

The most important point in the restoration of *Psammolepis* is the orientation of the branchials; as determined by their arching. In the figures by Gross [(²), Pl. 1; Figs. 1 and 5] the lateral free portions of the branchials protrude horizontally at the limit between the dorsal and ventral sides of the trunk. A similar view on the position of the branchials is maintained by A. Heintz and V. Paul. However, the material at my disposal shows that in all Psammosteids, except *Drepanaspis* and *Schizosteus*, the branchials are considerably arched along their whole surface, the dorsal as well as the ventral one. Thus, there can be no such direct line as that formed in Gross' figures by the dorsal surface (in transverse section). Therefore, the branchials, when placed on each side of the trunk, inevitably curve downwards (being concave ventrally). How far downwards shows their worn-off lateral margin which was therefore in contact with the bottom. The strongly convex and worn-off middle of the ventral plate must also have been in contact with the bottom. All this was possible only when the animal took such a position upon the substratum, as the one shown in Fig. 2.

What was the reason of this position? *Drepanaspis* lived, as it seems; in calm waters of a lagoon. Its descendants passed to a fluviatile life. In connexion with this; their branchials began to broaden and to curve downwards. Having come into contact with the substratum, they began to function not only as gliding planes; but also as anchors permittting the animals to hold against the current. In this position, the tail was raised, but did not put great resistance to the current; since it was compressed vertically. The flat anterior part of the fish was pressed to the bottom. This was strengthened by the sucking action of the vacuity under the branchials. In this position the fish stood against the current, filtering more or less passively ooze with its nutritive matter through the intestines: the structure of the Psammosteid oral plates (*Drepanaspis*) permits no thought of an active mode of feeding.

In later Psammosteids of the genus *Psammosteus* the branchials became strongly shortened, so that only narrow supports remained; protruding obliquely backwards and curved downwards. Parallel with this; the median plates became thinner; the scales smaller; the whole armour lighter, the animal more mobile. Yet, the habit to stay at the bottom remained. This is indicated by a still greater abrasion of the outer margins, and especially of the ends of the branchials.

Our restoration of *Psammolepis* is a first attempt, and doubtless the future finds will bring with them considerable corrections. Still I think that the chief structural characters of this genus have been reproduced correctly.

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HELMINTHOLOGY

ON THE DECIPHERMENT OF THE BIOLOGICAL CYCLE IN *MONIEZIA BENEDENI* (MONIEZ, 1879), TAPEWORM PARASITIC OF CATTLE AND OTHER DOMESTIC ANIMALS

By V. A. POTEMKINA

(Communicated by K. I. Skrjabin, Member of the Academy, 18. IV. 1943)

Soon after the developmental cycle of *Moniezia expansa* (Rudolphi, 1810) had been deciphered by Stunkard (1937) who also discovered the remarkable fact of this parasite using as intermediate hosts small soil ticks—oribates, studies devoted to the biology of *Anoplocephalidae* were started on a large scale at the K. I. Skrjabin All-Union Helminthological Institute.

Bashkirova has succeeded in deciphering the developmental cycle of all three species of tapeworms parasitic of the horse in our geographical zone (*Anoplocephala perfoliata*, *Anoplocephala magna* and *Paranoplocephala mamillana*).

The author of the present article was able to investigate the cycle of development of the tapeworm of ruminants, *Thysaniezia ovilla*; she succeeded in establishing under rigorous experimental conditions that in our geographical zone the main intermediate hosts of the tapeworm *Moniezia expansa* are two species of soil ticks: *Galumna obvius* (from the family *Galumnidae*) and *Scheloribates laevigatus* (from the fam. *Ceratozetidae*).

The present work is confined to the decipherment of the developmental cycle of *Moniezia benedeni* whose biology has not yet been the object of investigation.

More than 13000 ticks of the group *Oribatei* were infested with eggs of *Moniezia benedeni*.

In order to find out whether the larval form of *Moniezia benedeni* is capable of developing in the body of a tick, we dissected at various terms about 2000 oribates; among which representatives of the families *Galumnidae* and *Ceratozetidae* were prevalent.

After three days upon infestation of the ticks with *M. benedeni* eggs, body cavities of the ticks *Scheloribates laevigatus* and *Galumna obvius* were found to contain mobile oncospheres 0.016 and 0.020 mm across, armed with six embryonic hooks, the length of which attained 0.0095 mm.

Towards the 25th day, at a temperature of 19° C the oncospheres showed an increase in size, reaching 0.075×0.096 mm in diameter; they are filled with rather large homogeneous cells. The embryonic hooks have lost their regular arrangement.

According as the development of the larvae advanced, they grew considerably and underwent a number of changes. Thus, for instance, at the age of 35 days at a temperature of 17—19° C, the cross-section of a larva was 0.086×0.075 mm; by the 55—70th day larvae of oval shape reached a size of 0.092×0.071 mm across.

The body was found to be filled with homogeneous cells which at one of the poles passed into the appendage; the embryonic hooks are disorderly distributed at the tip of the appendage. At a temperature of 26° C, 60 days upon infestation of the ticks; and by the 100—105th day at 19° C, there were recorded larvae of

Fig. 1. *Moniezia benedeni*, larva of the «elongated» stage

the «elongated» stage, 0.540 mm in length against a maximum width of 0.240 mm. In the centre of the wide part one could already trace the configuration of the suckers, as well as the origination of some calcareous bodies. The embryonic hooks were arranged in the caudal part.

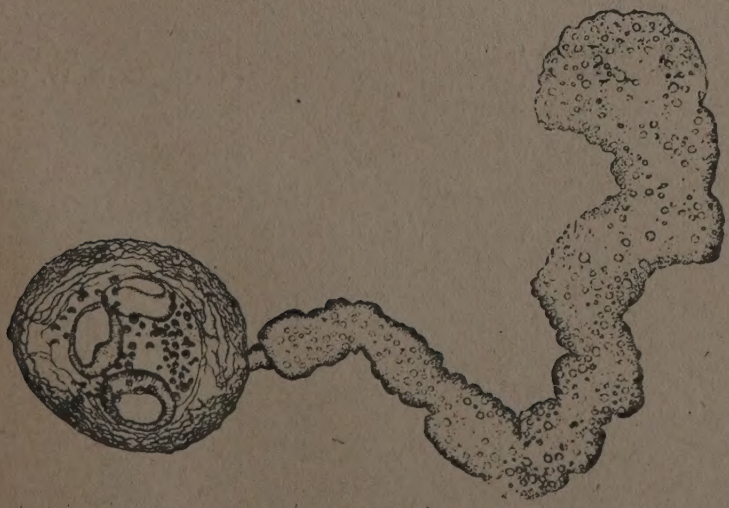

Fig. 2. *Moniezia benedeni*, cysticercoid

After 70—80 days, at a temperature of 26° C, and towards the 105th or 120th day, at 17—19° C, we used to observe larvae of the «segmented» stage, consis-

ting of three segments. The anterior oval part was now 0.320 mm long against a maximum width of 0.160 mm. The outlines of the suckers were clear-cut, oval in shape; 0.070×0.048 mm across; there was a group of calcareous bodies. The central part of the larva was the shape of an oval capsule, 0.190×0.180 mm across, covered with a solid membrane passing into the caudal appendage which is filled with delicate cells and shows a fine-granular structure. The length of the caudal appendage is 0.525 mm, its maximum width being 0.060 mm. Towards the 85th day at a temperature of 26° C we extracted from the body cavity of the ticks larvae which were about to achieve the invagination of their cephalic end.

After 110—130 days, the temperature ranging within 20—23° C, the cysticercoid formed was 0.145 × 0.125 mm across; four oval suckers 0.040 × 0.080 mm across could distinctly be seen within it, as well as a large number of calcareous bodies amply distributed between the scolex and the walls of the cyst. The walls of the latter are solid, about 0.006 mm thick, consisting of several layers. The caudal appendage reaches 0.5 mm in length and 0.07 in width. Two pairs of embryonic hooks are preserved at its tip.

With lower temperatures (say, 17—19°C) the cysticercoids reached the stage described above not until the 150—170th day.

The development of the cysticercoids *Moniezia benedeni* is affected not only by temperature, but by the specific composition of their intermediate hosts; as well. Thus, for example, the cysticercoids *Moniezia benedeni* extracted from the body cavity of ticks of the family *Ceratozetidae* did not exceed 0.16—0.17 mm in diameter, while the cysticercoid in ticks of the family *Galumnidae* reached 0.18—0.19 mm.

At a temperature of 26° C, *ceteris paribus*, the complete developmental cycle of cysticercoids in a tick requires 100—120 days to be achieved. At a temperature of 17—19° C the period required was as long as 140—150 days. And during the winter season under similar temperature conditions the development of cysticercoids took up to 170 days.

Thus, our experiments enabled us to decipher the developmental cycle of *M. benedeni* and to find out that this tapeworm develops under participation of intermediate hosts. These are soil ticks, oribates of two species: *Scheloribates laevigatus* from the fam. *Ceratozetidae*, and *Galumna obvius* from the fam. *Galumnidae*.

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